CS622

(Deterministic) Finite Automata

Monday, January 29, 2024

UMass Boston Computer Science

Announcements

- **HW**
  - Weekly; in/out Mon noon
    - HW 0 in, HW 1 out
  - ~3-4 questions, Paper-and-pencil proofs (no programming)
  - Discussing with classmates ok
  - Final answers written up and submitted individually

- **Lectures**
  - Slides posted
  - Closely follow the listed textbook chapters
- **Office Hours**
  - **Wed 11:30-1pm** (in person, McCormack 3rd floor, Rm 201)
  - **Fri 11:30-1pm** (zoom, access link from blackboard)
  - Let me know in advance if possible, but drop-ins also fine
  - TAs TBD

Last Time: How Mathematics Works

Today:
- "Facts" can have many different "shapes"!
- How do we USE known facts?
- How can we PROVE new facts?

Mathematician
(or student)

It's not easy to create the next level …
Preciseness is important

Proofs = Figure out how to (precisely) fit known "facts" together

More **Theorems**

More **Axioms**

More **Definitions**

Theorem

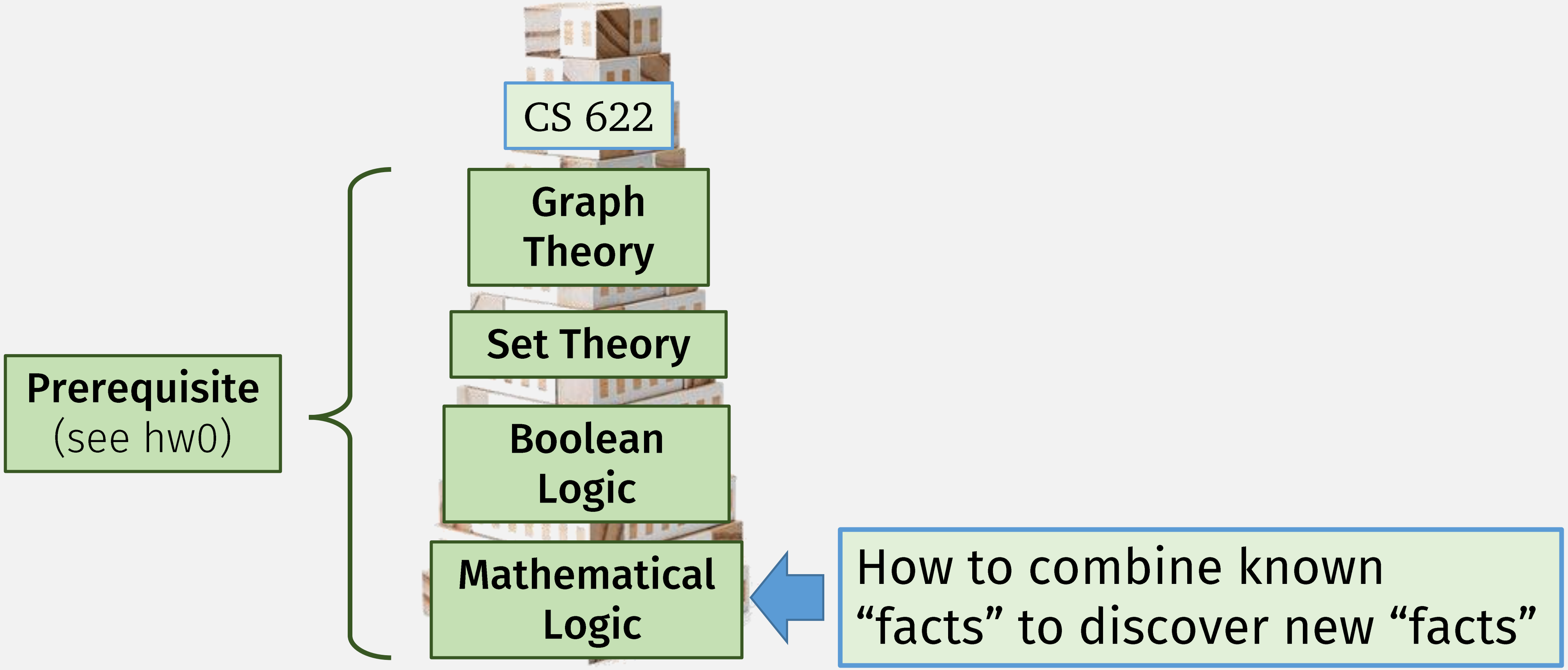
Theorem

Axioms

Definitions

"facts"

Last Time: How **CS 622** Works



CS 622

Graph Theory

Set Theory

Boolean Logic

Mathematical Logic

Prerequisite
(see hw0)

How to combine known “facts” to discover new “facts”

Mathematical Logic Operators

- **Conjunction** (AND, $\land$)
- **Disjunction** (OR, $\lor$)
- **Negation** (NOT, $\neg$)
- **Implication** (IF-THEN, $\Rightarrow$, $\rightarrow$)

…

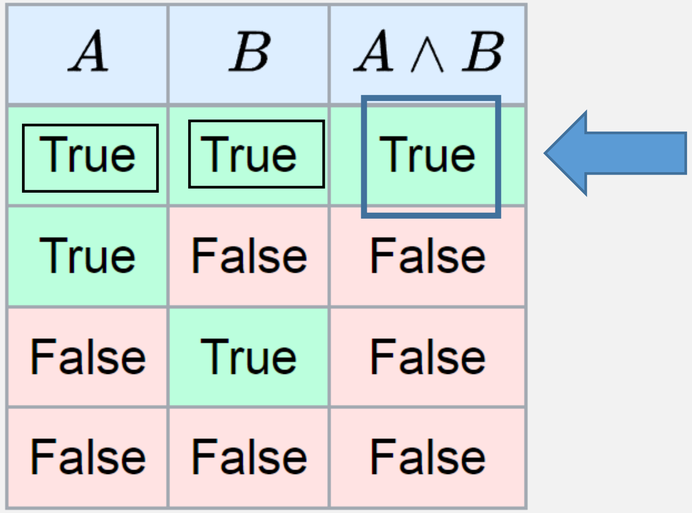
This semester:

Must understand difference between **Using** vs **Proving** a mathematical statement!

Mathematical Statements: **AND**

Using:

- If we know $A \land B$ is **TRUE**, what do we know about A and B individually?
  - $\boxed{A}$ is **TRUE**, and
  - $\boxed{B}$ is **TRUE**



A	B	$A \land B$
True	True	True
True	False	False
False	True	False
False	False	False

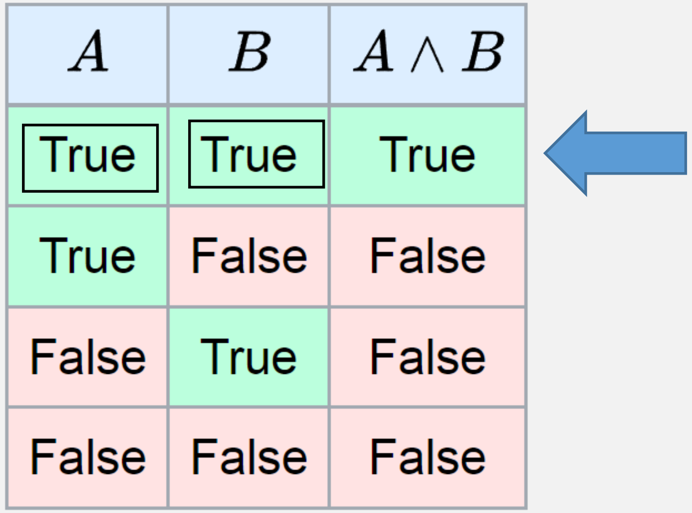
Mathematical Statements: **AND**

Using:

- If we know $A \wedge B$ is $\mathbf{TRUE}$,
  what do we know about A and B individually?
  - A is $\mathbf{TRUE}$, and
  - B is $\mathbf{TRUE}$

Proving:

- To prove $A \wedge B$ is $\mathbf{TRUE}$:
  - Prove $\boxed{A}$ is $\mathbf{TRUE}$, and
  - Prove $\boxed{B}$ is $\mathbf{TRUE}$

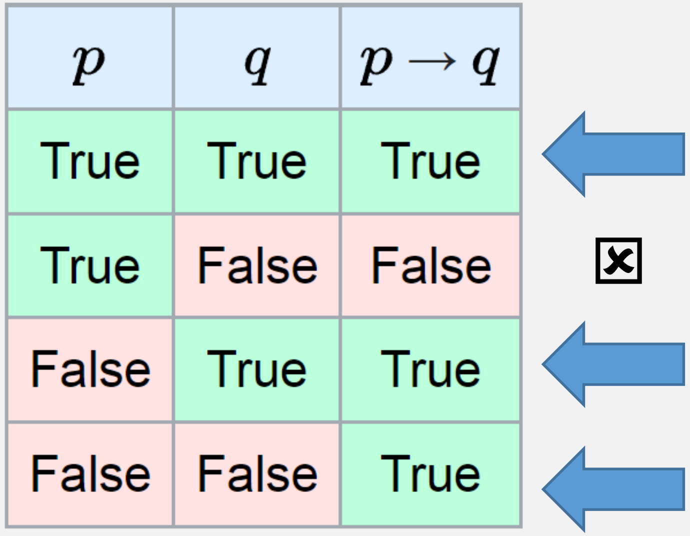


A	B	$A \wedge B$
$\boxed{\text{True}}$	$\boxed{\text{True}}$	True
True	False	False
False	True	False
False	False	False

Mathematical Statements: **IF-THEN**

Using:

- If we know $P \rightarrow Q$ is $\mathbf{TRUE}$,
  what do we know about P and Q individually?
  - Either P is $\mathbf{FALSE}$, or
  - If we **prove** P is $\mathbf{TRUE}$, then Q is $\mathbf{TRUE}$ (**modus ponens**)



p	q	$p \rightarrow q$	
True	True	True	←
True	False	False	☒
False	True	True	←
False	False	True	←

Proving:

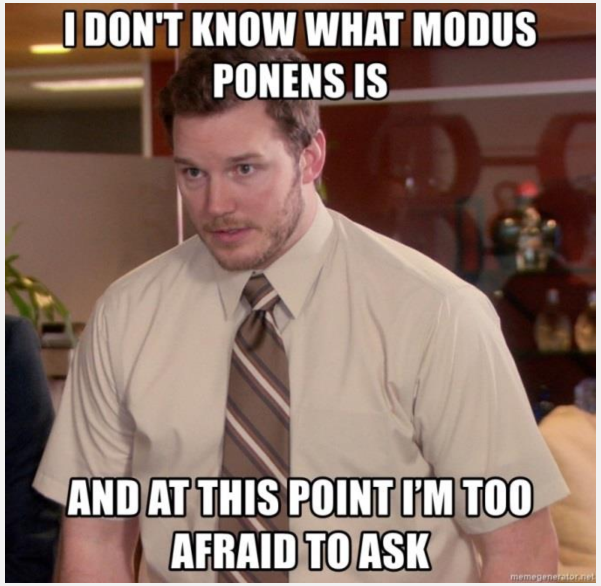
Using an **IF-THEN** statement:
The “Modus Ponens” Inference Rule

Premises (if these statements are true)

- If P then Q
- P is TRUE

Conclusion (then we can say that this is also true)

- Q must also be TRUE


I DON'T KNOW WHAT MODUS PONENS IS
AND AT THIS POINT I'M TOO AFRAID TO ASK
memegenerator.net

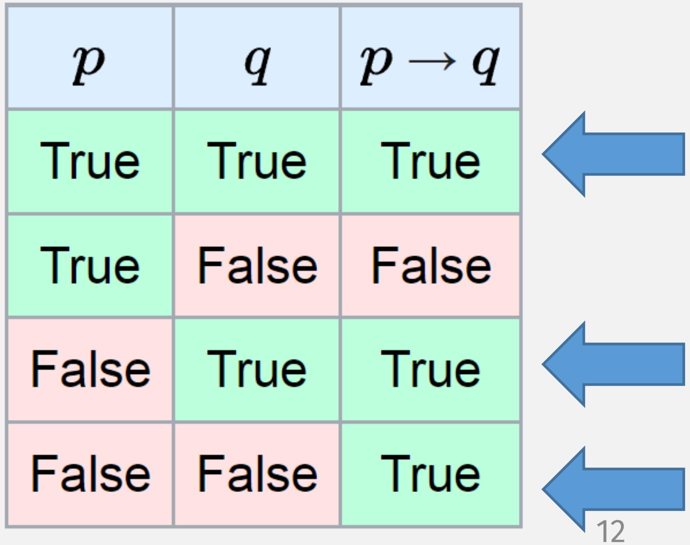
Mathematical Logic Operators: **IF-THEN**

Using:

- If we know $P \to Q$ is TRUE,
  what do we know about P and Q individually?
  - Either P is FALSE, or
  - If we **prove** P is TRUE, then Q is TRUE (**modus ponens**)

Proving:

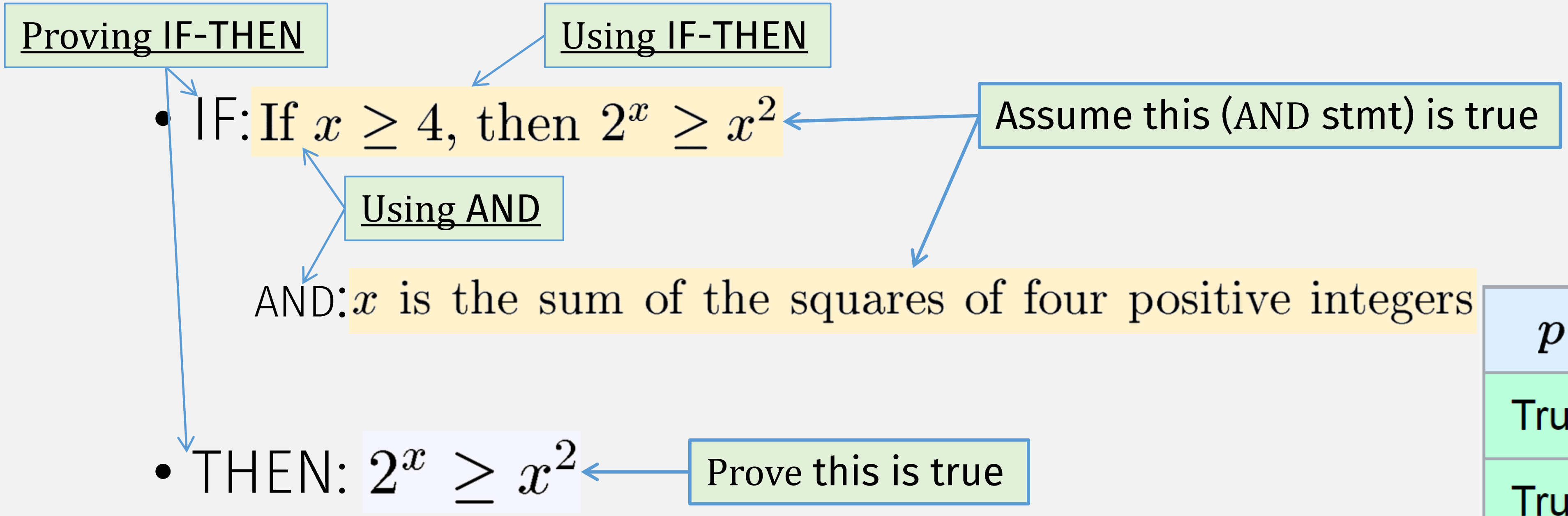
- To prove $P \to Q$ is TRUE:
  - Either **Prove** P is FALSE (usu. hard or impossible), or
  - **Assume** (not prove!) P is TRUE,
    then **prove** Q is TRUE



p	q	$p \to q$
True	True	True
True	False	False
False	True	True
False	False	True

Example: Proving an IF-THEN Statement

Prove the following:



Proving IF-THEN

Using IF-THEN

- IF: If $x \geq 4$, then $2^x \geq x^2$

Assume this (AND stmt) is true

Using AND

AND: x is the sum of the squares of four positive integers

- THEN: $2^x \geq x^2$

Prove this is true

Proving:

To prove $P \rightarrow Q$ is TRUE:

Either Prove P is FALSE (usu. hard or impossible), or

Assume (not prove!) P is TRUE, then prove Q is TRUE

p	q	$p \rightarrow q$
True	True	True
True	False	False
False	True	True
False	False	True

Example: Proving an IF-THEN Statement

Prove: IF If $x \geq 4$, then $2^x \geq x^2$ AND x is the sum of the squares of four positive integers
THEN $2^x \geq x^2$

Proof:

Statement	Justification
1. $x = a^2 + b^2 + c^2 + d^2$	1. Assumption (IF part of IF-THEN)
2. $a \geq 1;\ b \geq 1;\ c \geq 1;\ d \geq 1$	2. Assumption (IF part of IF-THEN)
5. If $x \geq 4$, then $2^x \geq x^2$	5. Assumption (IF part of IF-THEN)
6. $2^x \geq x^2$	

Statements / Justifications Table

Example: Proving an IF-THEN Statement

Prove: IF If $x \geq 4$, then $2^x \geq x^2$ AND x is the sum of the squares of four positive integers
THEN $2^x \geq x^2$

Proof:

Statement	Justification
1. $x = a^2 + b^2 + c^2 + d^2$	1. Assumption (IF part of IF-THEN)
2. $a \geq 1;\ b \geq 1;\ c \geq 1;\ d \geq 1$	2. Assumption (IF part of IF-THEN)
3. $a^2 \geq 1;\ b^2 \geq 1;\ c^2 \geq 1;\ d^2 \geq 1$	3. By Stmt #2 & arithmetic laws
4. $x \geq 4$	4. Stmts #1, #3, and arithmetic
5. If $x \geq 4$, then $2^x \geq x^2$	5. Assumption (IF pa
6. $2^x \geq x^2$	6. Stmts #4 and #5

Modus Ponens

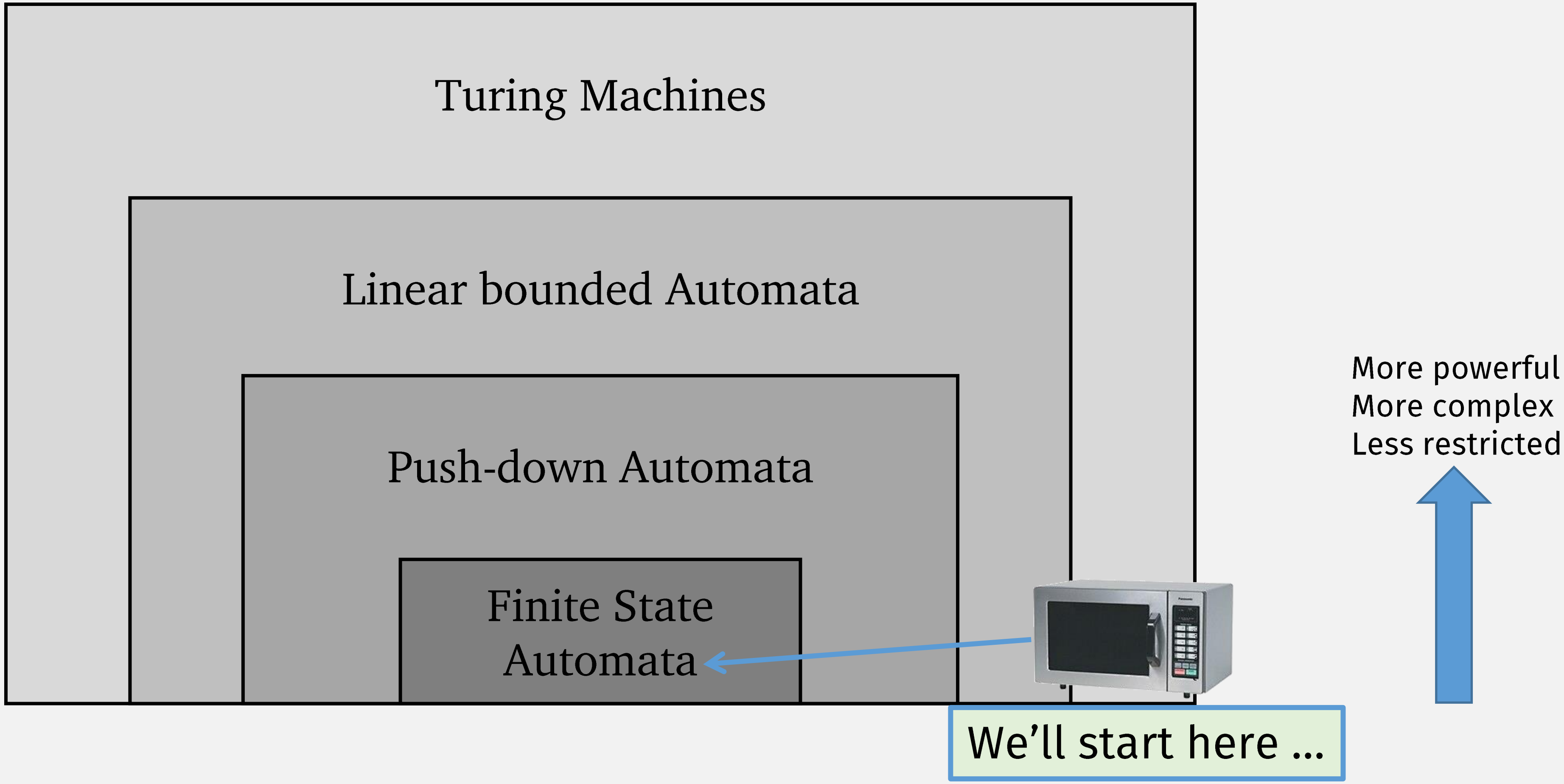
If we can prove these:

- If P then Q
- P

Then we've proved:

- Q

Last Time: Models of Computation Hierarchy



Turing Machines

Linear bounded Automata

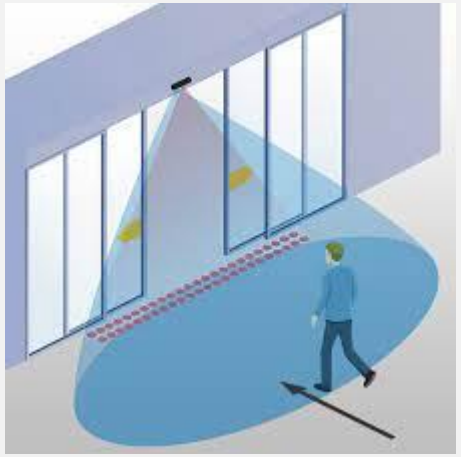
Push-down Automata

Finite State Automata

We’ll start here …

More powerful
More complex
Less restricted

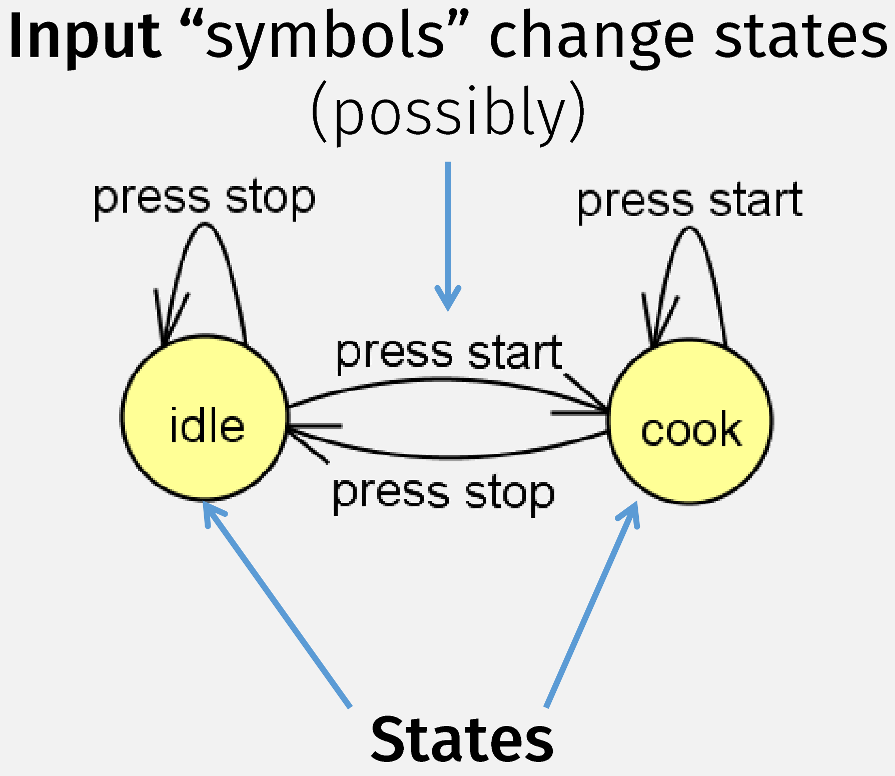
Finite Automata: “Simple” Computation / “Programs”



Finite Automata

- A **finite automata** or **finite state machine (FSM) …**
- … **computes** with a finite number of **states**

A Microwave Finite Automata

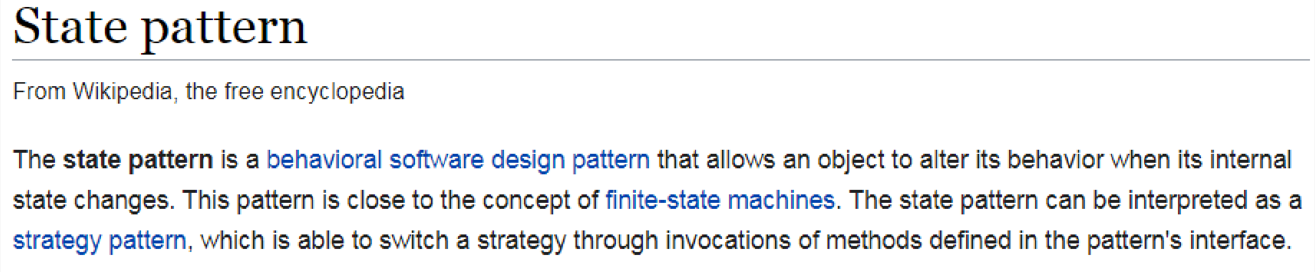


Input “symbols” change states

(possibly)

States

Finite Automata: Not Just for Microwaves



State pattern

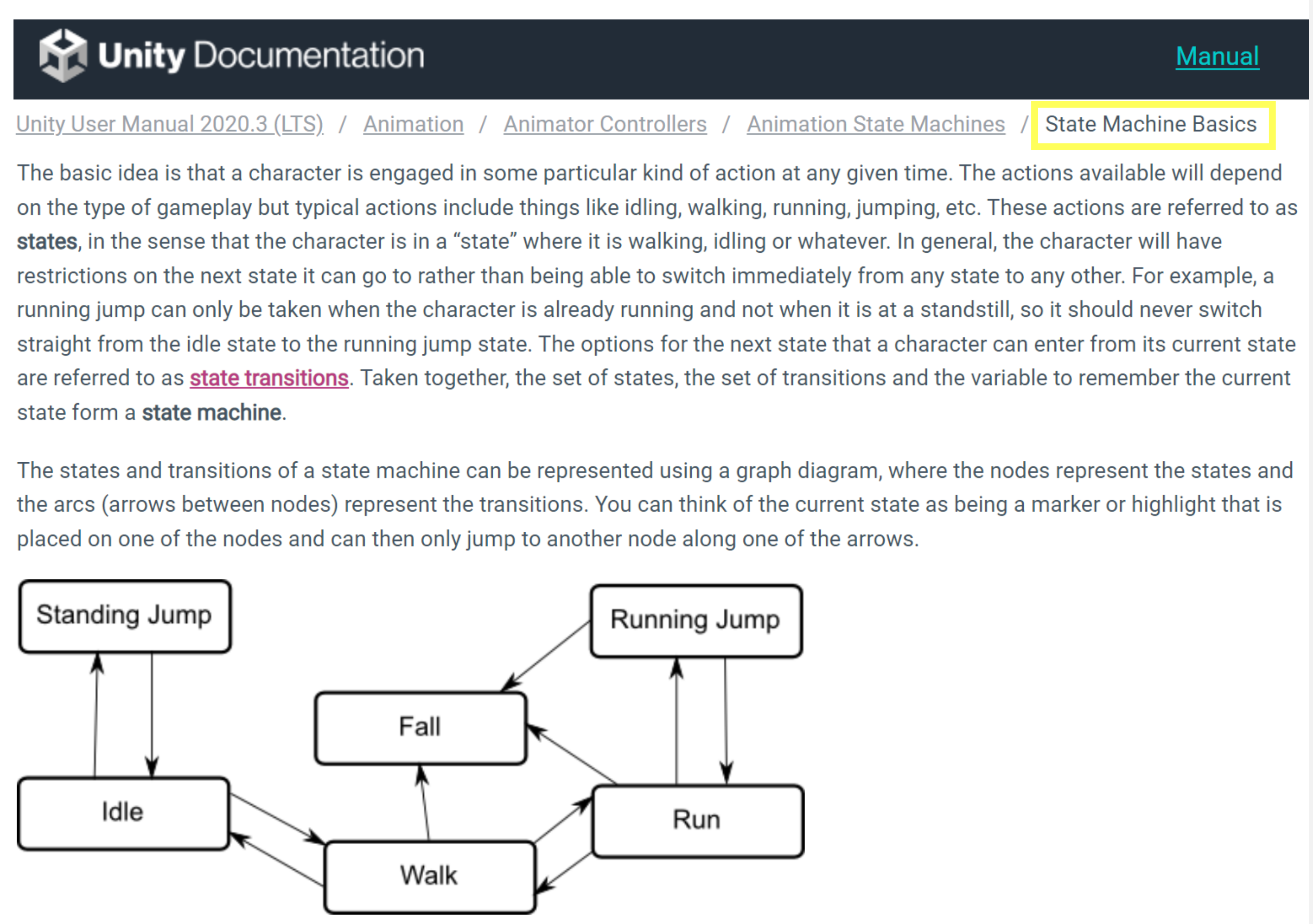
From Wikipedia, the free encyclopedia

The **state pattern** is a behavioral software design pattern that allows an object to alter its behavior when its internal state changes. This pattern is close to the concept of finite-state machines. The state pattern can be interpreted as a strategy pattern, which is able to switch a strategy through invocations of methods defined in the pattern's interface.

Finite Automata:
a common
programming pattern

(More powerful) **Computation Simulating other** (weaker) **Computation**
(a common theme this semester)

Video Games Love Finite Automata



Unity Documentation

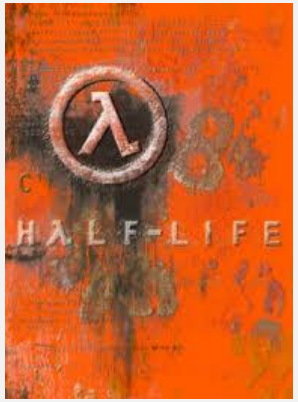
Manual

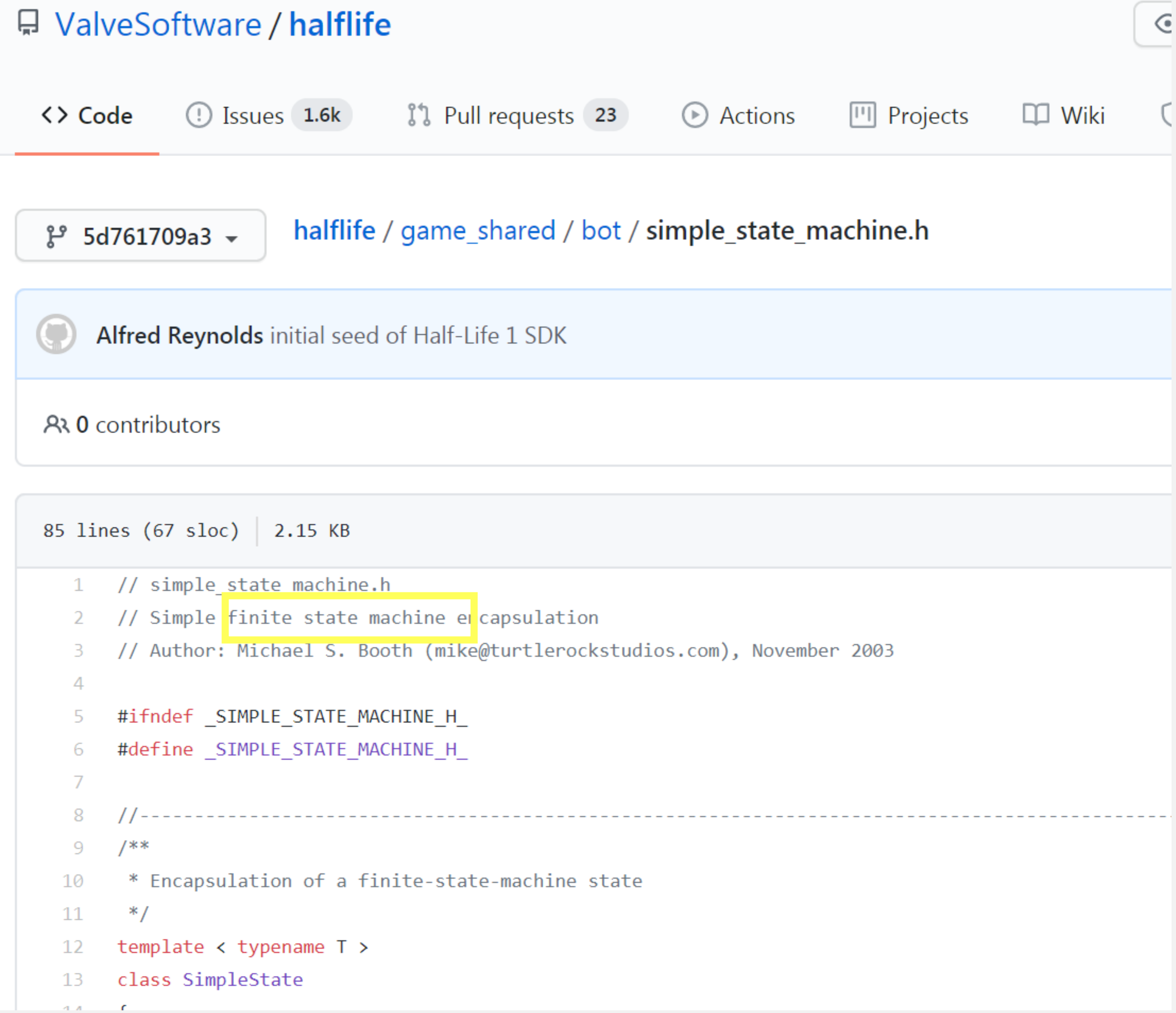
Unity User Manual 2020.3 (LTS) / Animation / Animator Controllers / Animation State Machines / State Machine Basics

The basic idea is that a character is engaged in some particular kind of action at any given time. The actions available will depend on the type of gameplay but typical actions include things like idling, walking, running, jumping, etc. These actions are referred to as **states**, in the sense that the character is in a "state" where it is walking, idling or whatever. In general, the character will have restrictions on the next state it can go to rather than being able to switch immediately from any state to any other. For example, a running jump can only be taken when the character is already running and not when it is at a standstill, so it should never switch straight from the idle state to the running jump state. The options for the next state that a character can enter from its current state are referred to as **state transitions**. Taken together, the set of states, the set of transitions and the variable to remember the current state form a **state machine**.

The states and transitions of a state machine can be represented using a graph diagram, where the nodes represent the states and the arcs (arrows between nodes) represent the transitions. You can think of the current state as being a marker or highlight that is placed on one of the nodes and can then only jump to another node along one of the arrows.

Finite Automata in Video Games





halflife / game_shared / bot / simple_state_machine.h

Alfred Reynolds initial seed of Half-Life 1 SDK

0 contributors

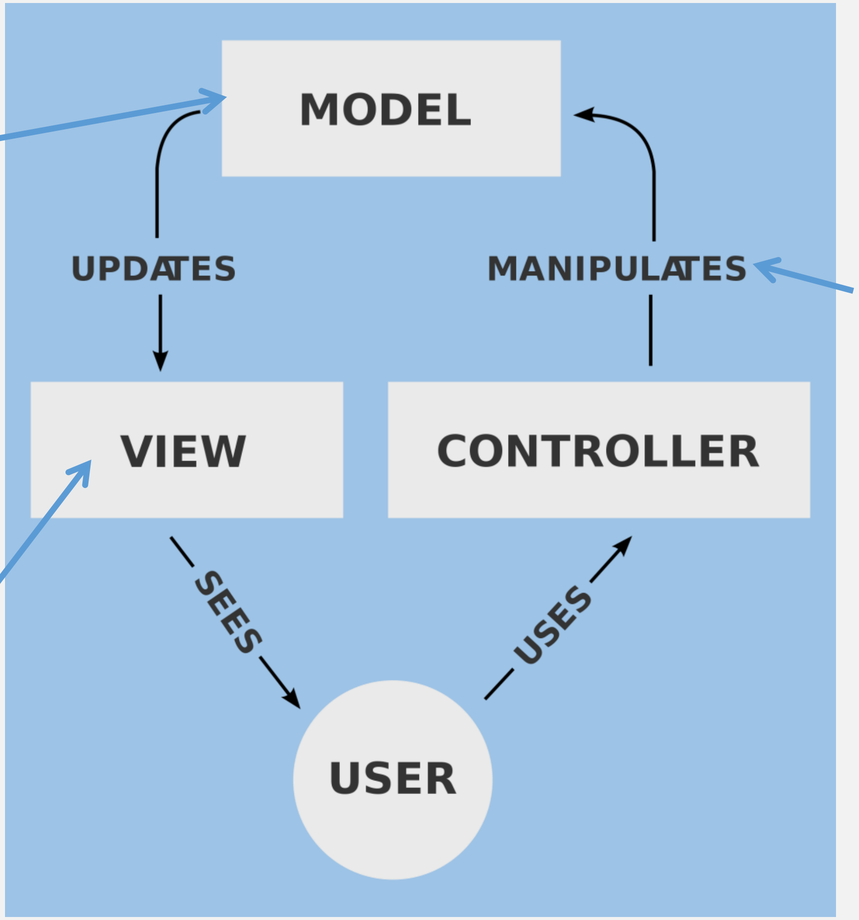
85 lines (67 sloc) | 2.15 KB

```
// simple_state_machine.h
// Simple finite state machine encapsulation
// Author: Michael S. Booth (mike@turtlerockstudios.com), November 2003

#ifndef _SIMPLE_STATE_MACHINE_H_
#define _SIMPLE_STATE_MACHINE_H_

//--------------------------------------------------------------------------------------------------------
/**
 * Encapsulation of a finite-state-machine state
 */
template < typename T >
class SimpleState
{
```

Model-view-controller (MVC) is an FSM



States

Input events change states

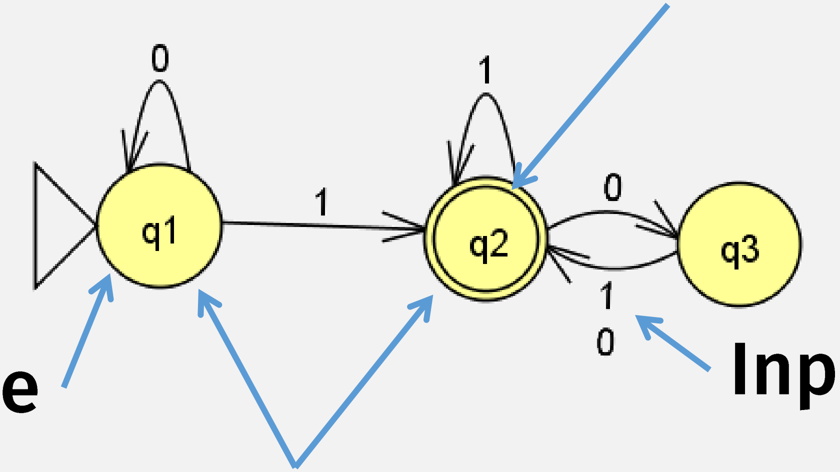
The **View** draws states

A Finite Automata is a “Program”

- A **very limited “program”** that uses **finite memory**
  - Actually, **only 1 “cell” of memory!**
  - **States** = the **possible things that can be written to memory**

- **Finite Automata has different representations:**
  - **Code** (wont use in this class)
  - ➢**State diagrams**

Finite Automata state diagram



Accept State

Start State

States

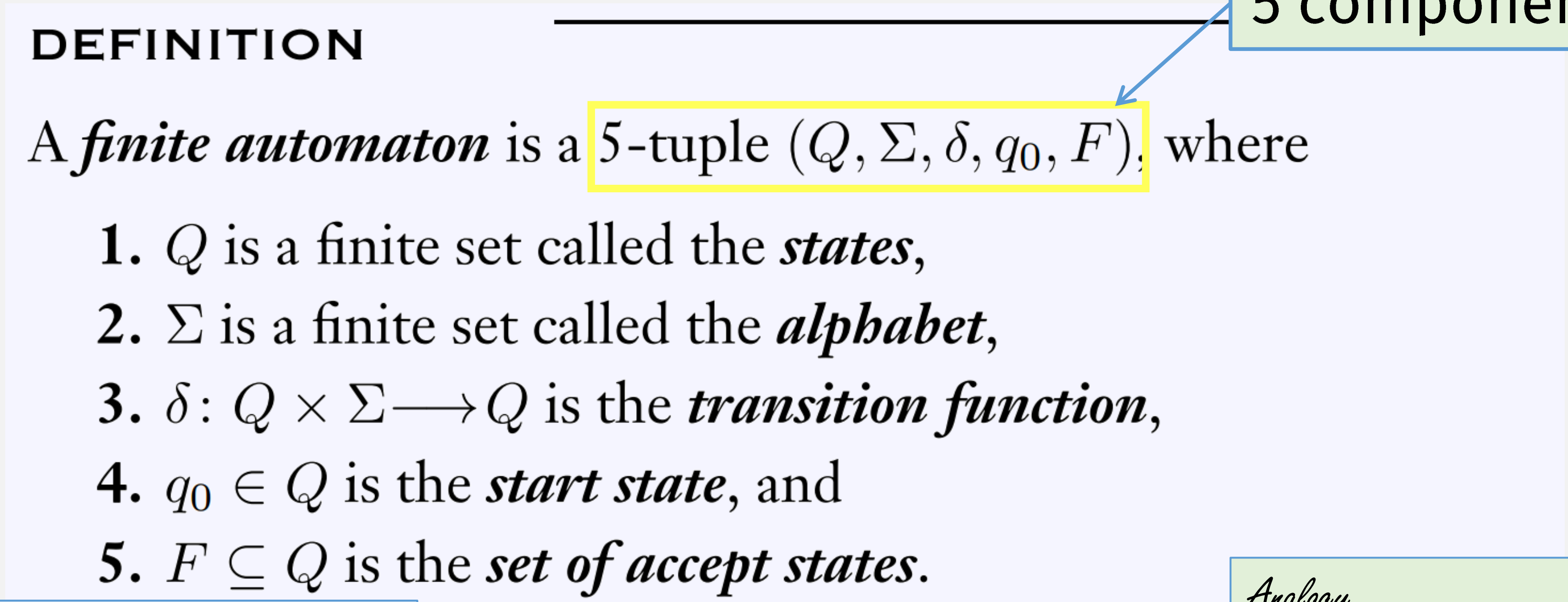
Inputs specify **state transitions**

A Finite Automata = a “Program”

- A very limited program with finite memory
  - Actually, only 1 “cell” of memory!
  - States = the possible things that can be written to memory

- Finite Automata has different representations:
  - Code
  - State diagrams
  - ➢Formal mathematical description (essentially like code)

Finite Automata: The Formal Definition



DEFINITION

A ***finite automaton*** is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

1. Q is a finite set called the ***states***,
2. Σ is a finite set called the ***alphabet***,
3. $\delta \colon Q \times \Sigma \longrightarrow Q$ is the ***transition function***,
4. $q_0 \in Q$ is the ***start state***, and
5. $F \subseteq Q$ is the ***set of accept states***.

5 components

This semester

Things in **bold** are **precise formal definitions.**
(remember them because they will appear frequently in hw, etc)

Analogy

This is the **"programming language" definition** for **finite automata "programs"**

Interlude: Sets and Sequences

- Both are: **mathematical objects that group other objects**
- **Members** of the group are called **elements**
- Can be: **empty, finite, or infinite**
- Can contain: **other sets or sequences**

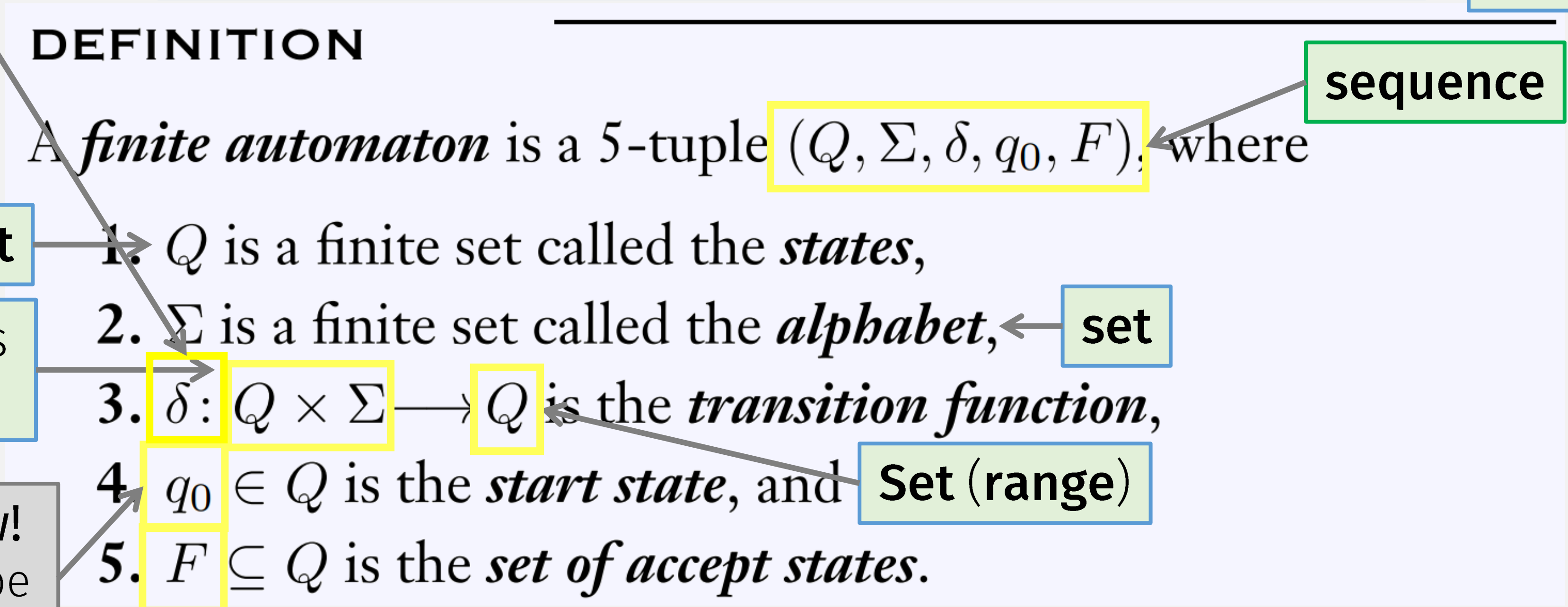
Sets	**Sequences**
Unordered	Ordered
Duplicates not allowed	Duplicates ok
Common notation: { }	Common notation: ( ), or **just commas**
Empty set denoted: ∅ or { }	**Empty sequence:** ( )
A **language** is a (possibly infinite) set of strings	A **tuple** is a finite sequence
	A **string** is a finite sequence of characters

Set or Sequence ?

A **function** is …

… a **set** of **pairs**
(1st of each pair **from domain**, 2nd **from range**)

… can write it in many ways: as a mapping, a table, …



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sequence

set

set

Set of pairs (**domain**)

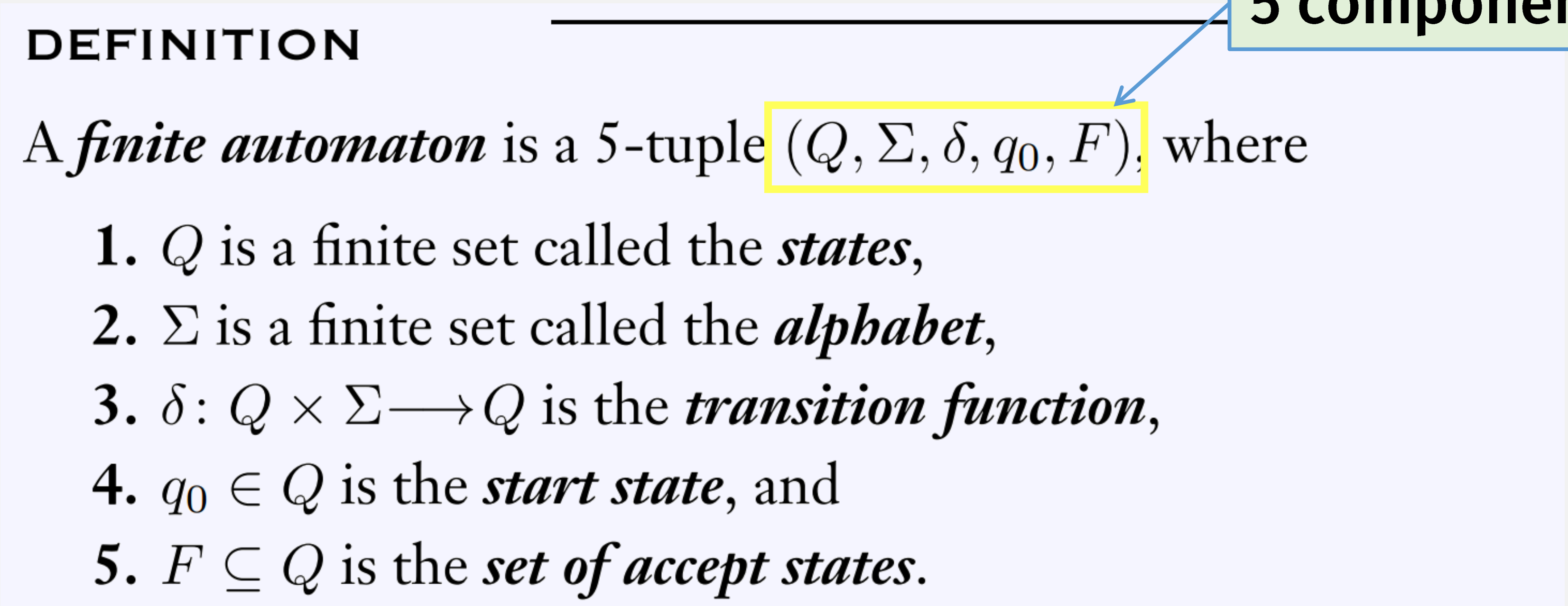
Set (**range**)

Don't know! (states can be anything)

set

A **pair** is … a **sequence** of 2 elements

Finite Automata: The Formal Definition



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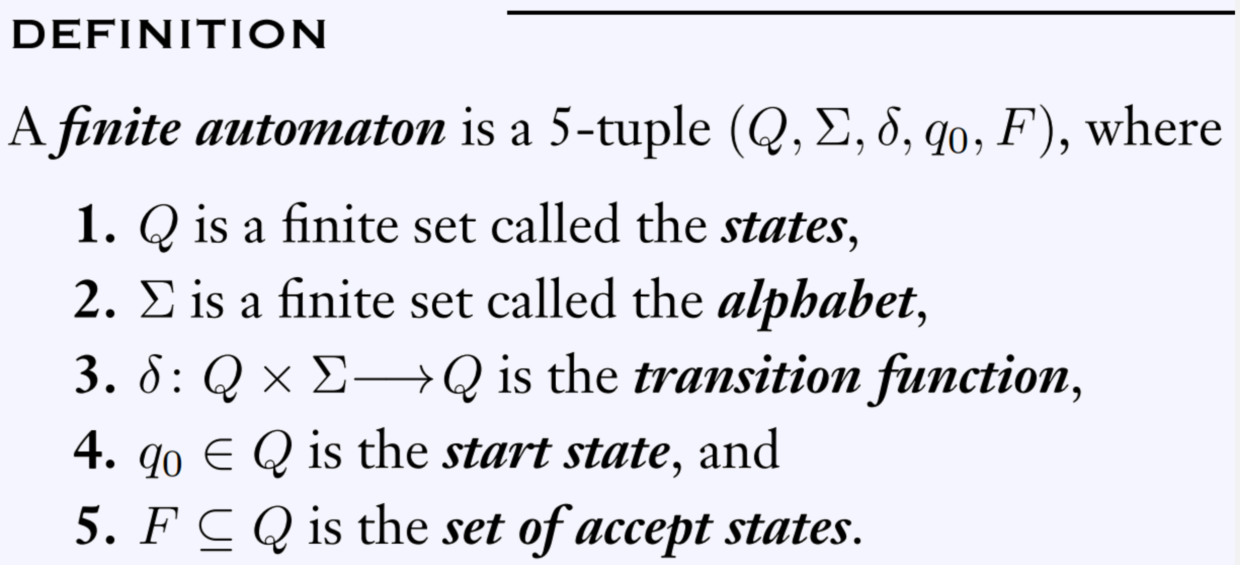
5 components



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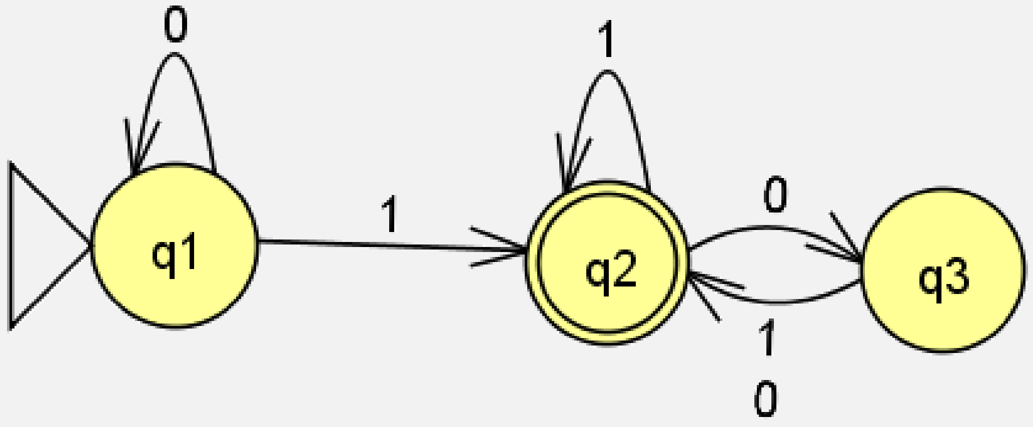
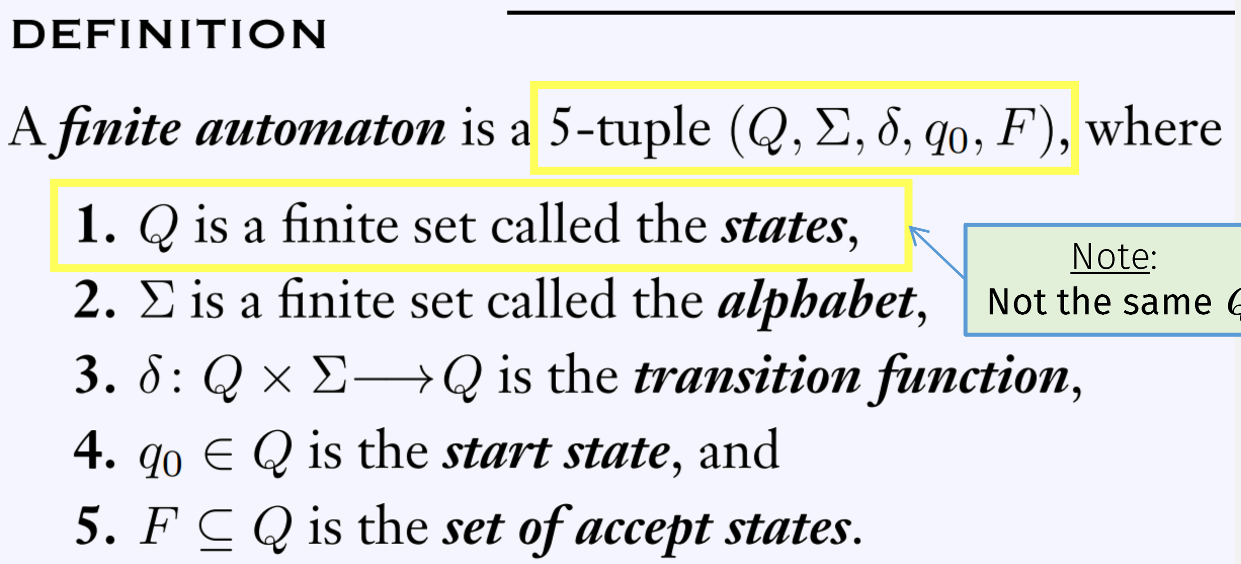


Example: as state diagram



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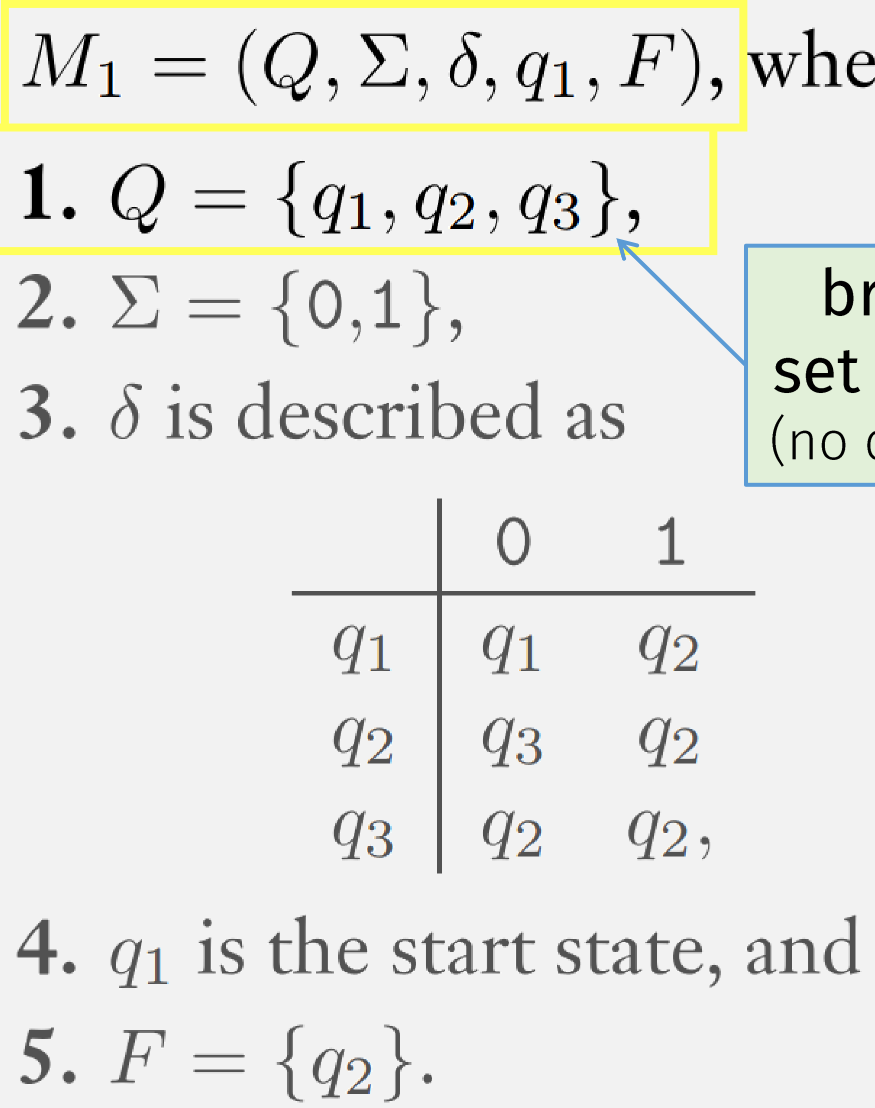
Note:
Not the same Q

Example: as formal description



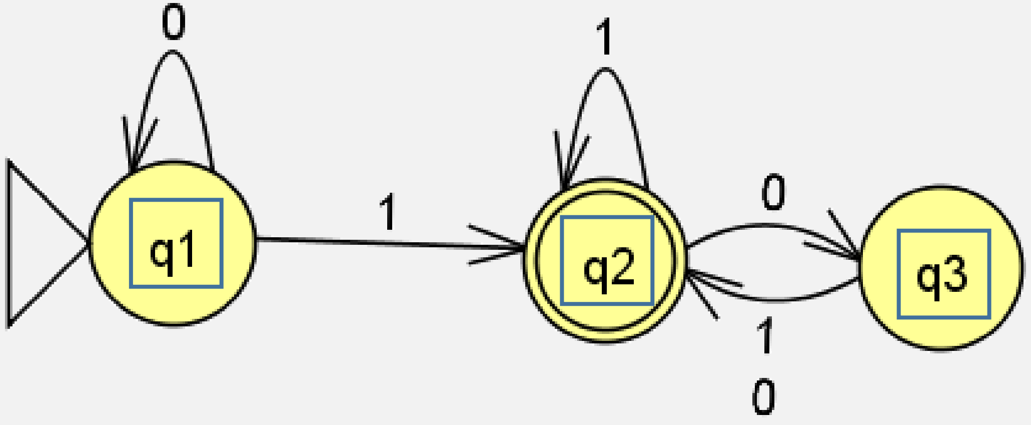
$M_1 = (Q, \Sigma, \delta, q_1, F)$, where

1. $Q = \{q_1, q_2, q_3\}$,
2. $\Sigma = \{0,1\}$,
3. δ is described as

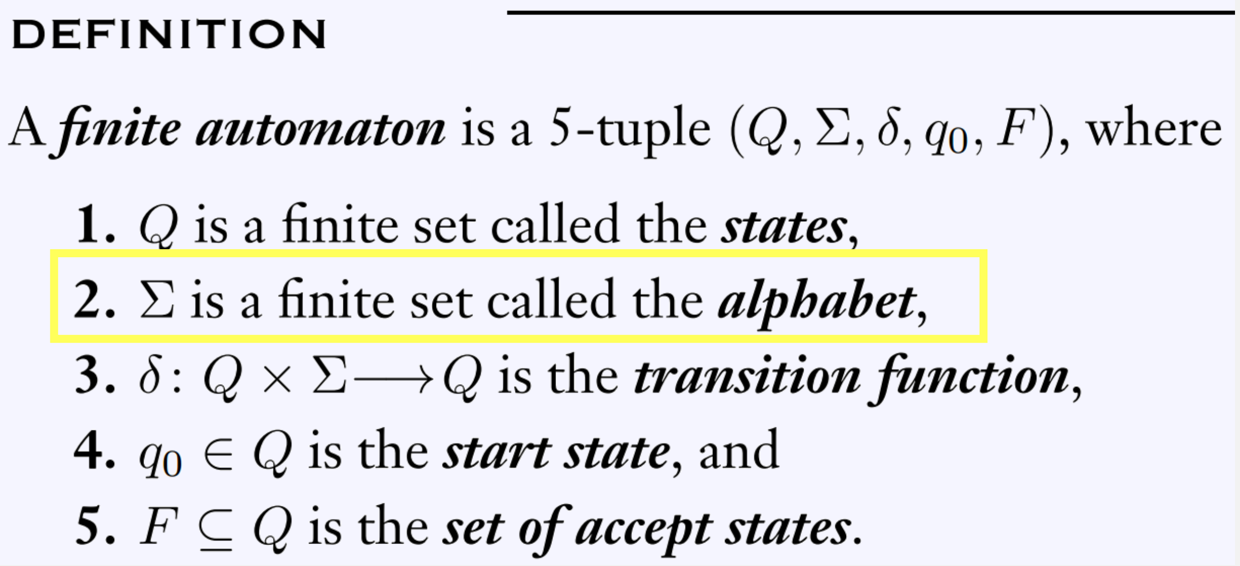
	0	1
q_1	q_1	q_2
q_2	q_3	q_2
q_3	q_2	q_2,

4. q_1 is the start state, and
5. $F = \{q_2\}$.

braces =
set notation
(no duplicates)



Example: as state diagram



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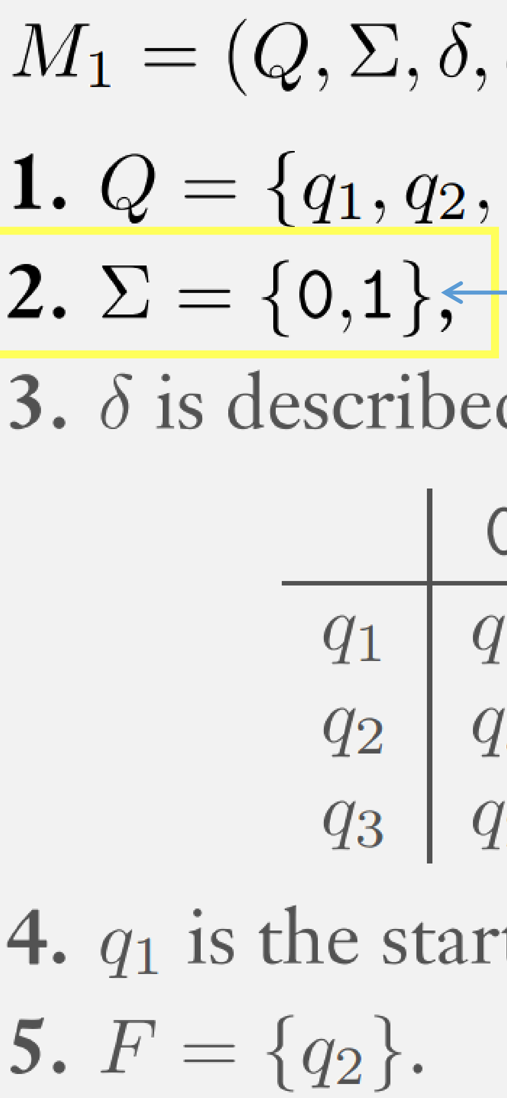
Example: as <u>formal description</u>



$M_1 = (Q, \Sigma, \delta, q_1, F)$, where

1. $Q = \{q_1, q_2, q_3\}$,
2. $\Sigma = \{0,1\}$, ← Possible inputs
3. δ is described as

	0	1
q_1	q_1	q_2
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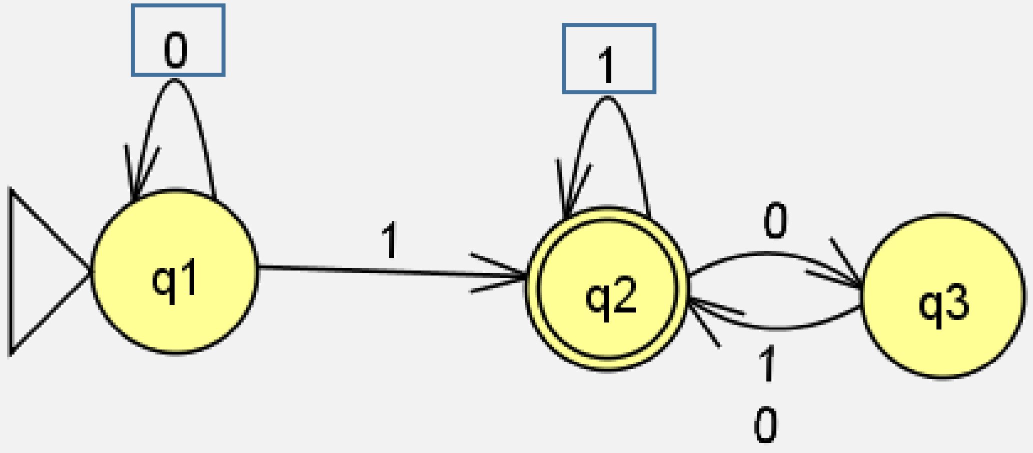
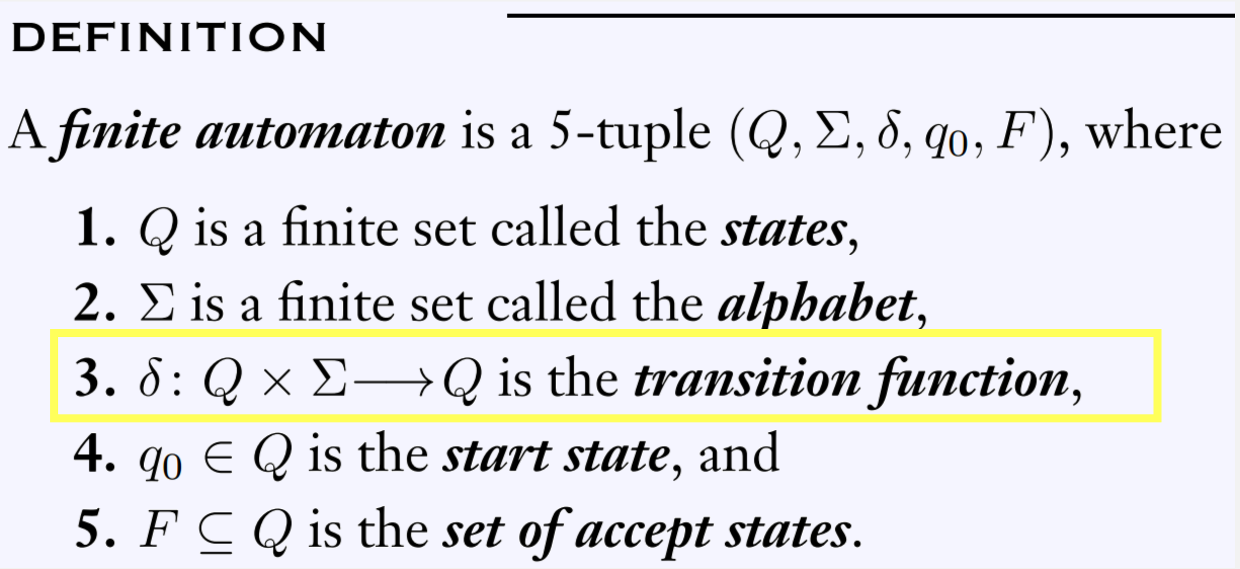


Example: as <u>state diagram</u>



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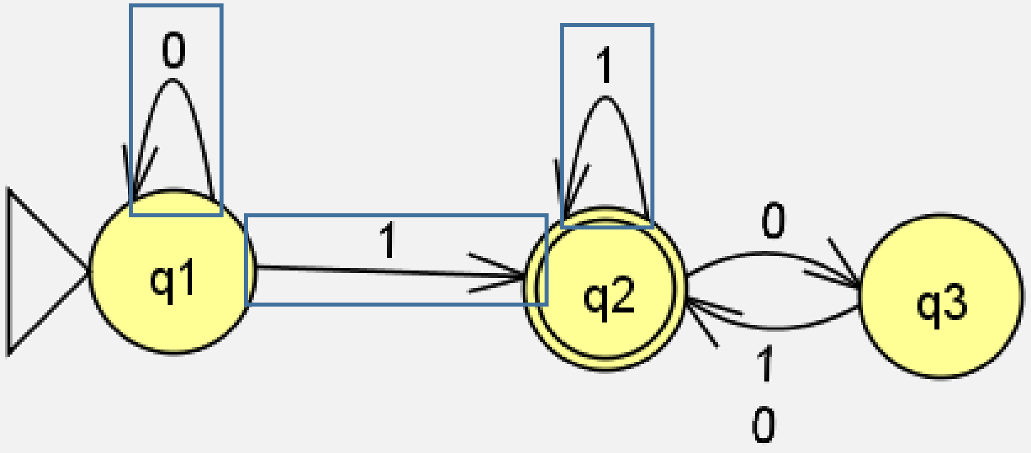

Example: as state diagram

Example: as formal description

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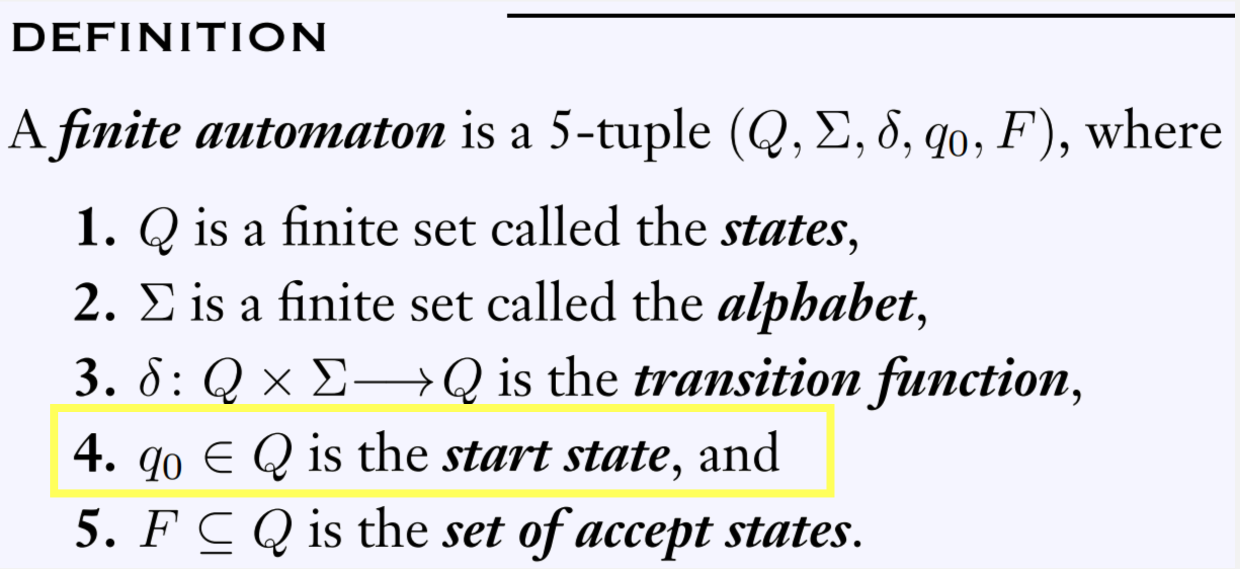
	0	1
q_1	q_1	q_2
q_2	q_3	q_2
q_3	q_2	q_2,

"And this is next input symbol"

"If in this state"

"Then go to this state"

4. q_1 is the start state, and
5. $F = \{q_2\}$.



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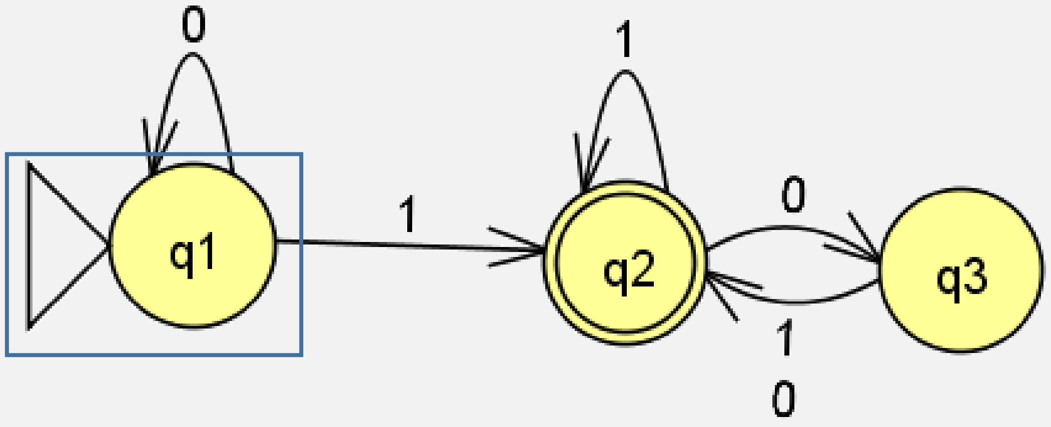

Example: as <u>state diagram</u>

Example: as <u>formal description</u>

$$M_1 = (Q, \Sigma, \delta, q_1, F)\text{, where}$$

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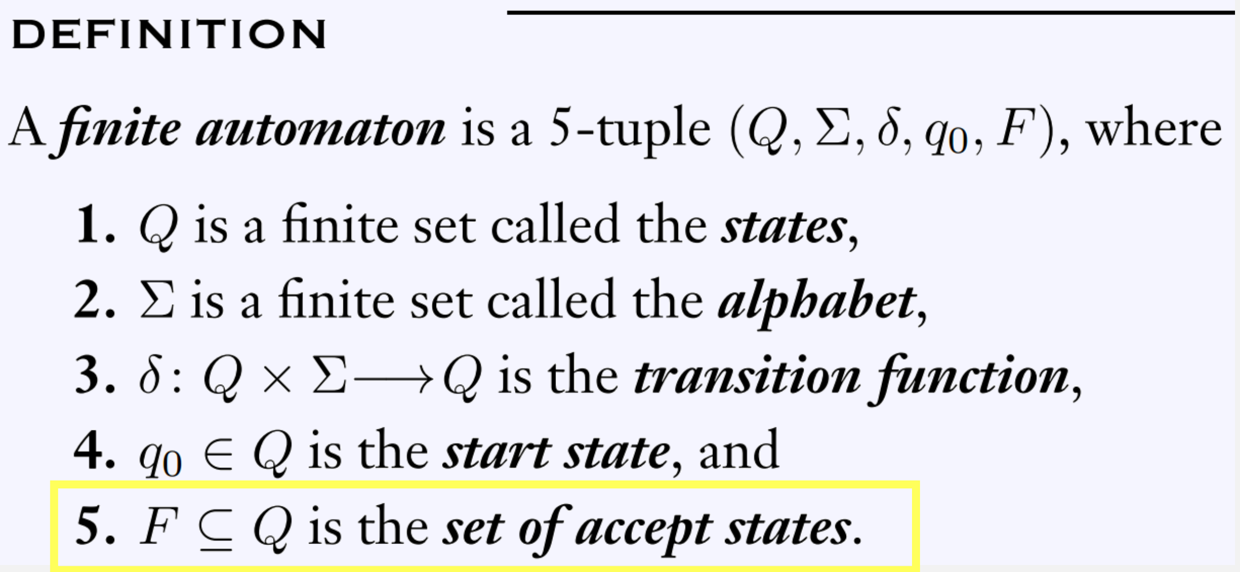
	0	1
q_1	q_1	q_2
q_2	q_3	q_2
q_3	q_2	q_2,

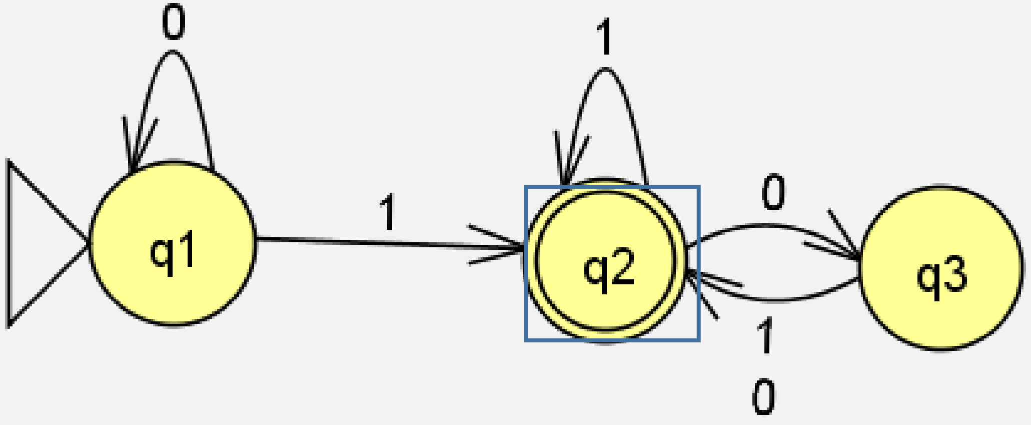
4. q_1 is the start state, and
5. $F = \{q_2\}$.



DEFINITION

A ***finite automaton*** is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

1. Q is a finite set called the ***states***,
2. Σ is a finite set called the ***alphabet***,
3. $\delta : Q \times \Sigma \longrightarrow Q$ is the ***transition function***,
4. $q_0 \in Q$ is the ***start state***, and
5. $F \subseteq Q$ is the ***set of accept states***.



Example: as <u>state diagram</u>

Example: as <u>formal description</u>

$$M_1 = (Q, \Sigma, \delta, q_1, F), \text{ where}$$

1. $Q = \{q_1, q_2, q_3\}$,
2. $\Sigma = \{0,1\}$,
3. δ is described as

	0	1
q_1	q_1	q_2
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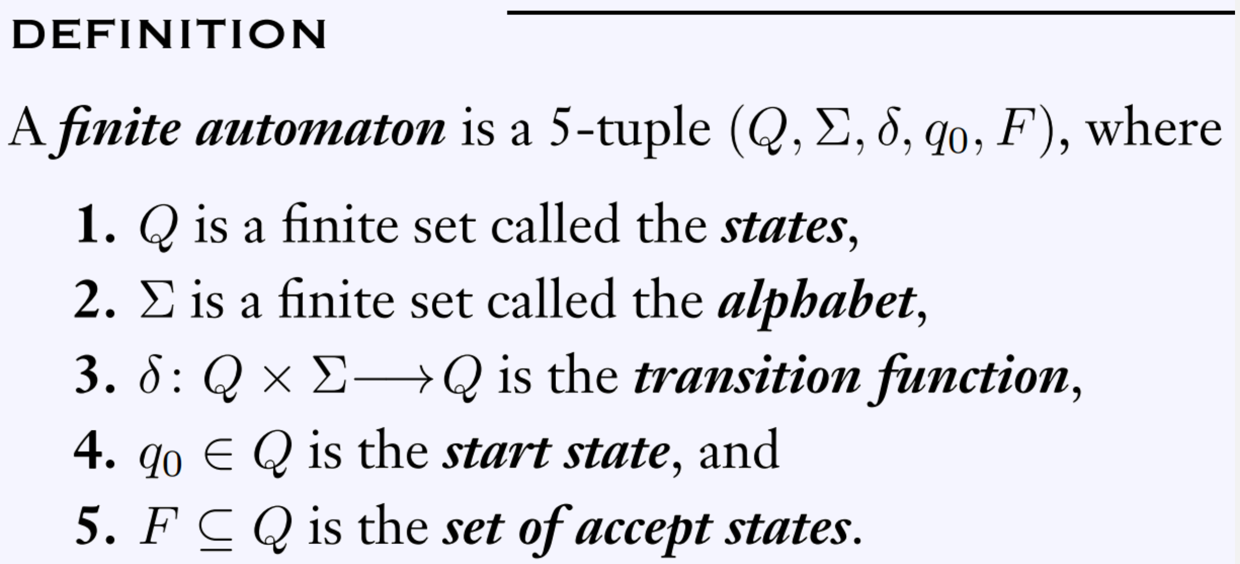
Programming Analogy



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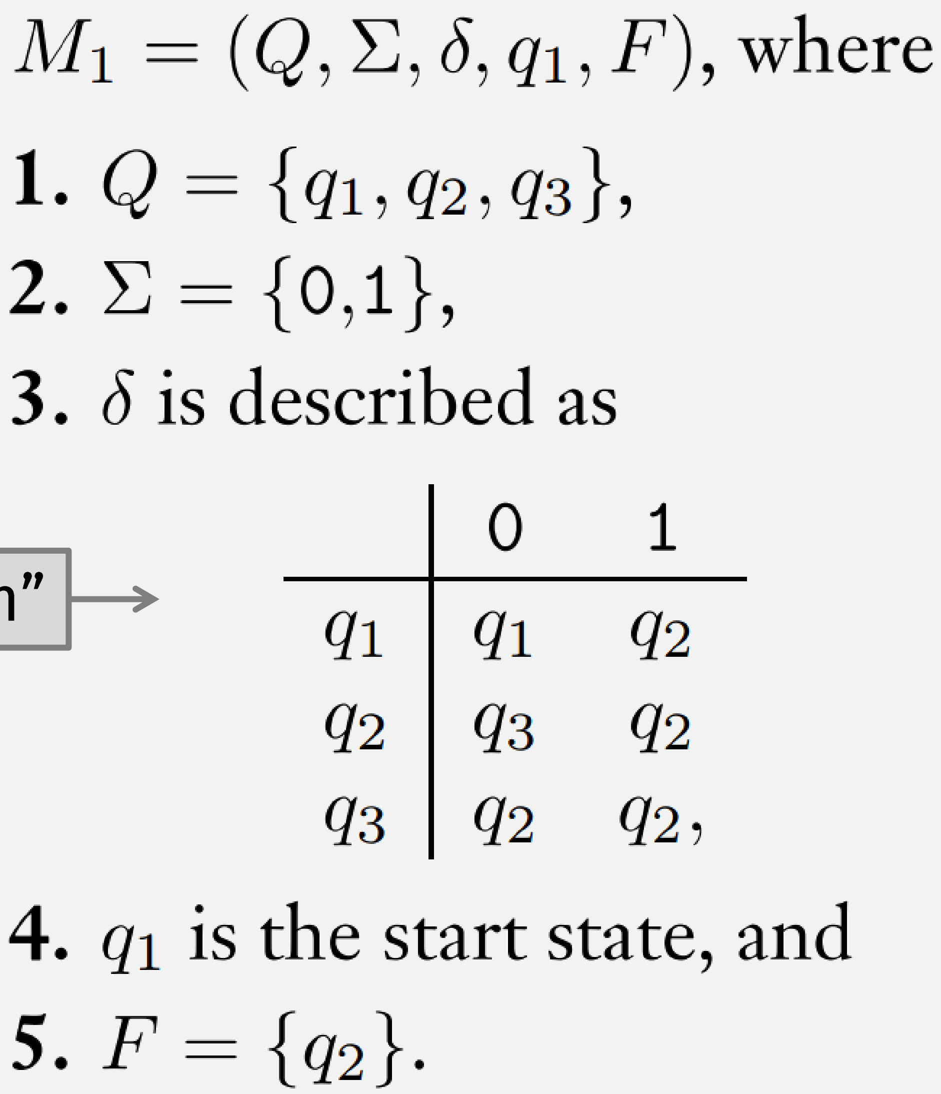
A "Programming Language"

This analogy is a way to help your intuition

But don't confuse with **formal definitions.**

Example: as formal description

A "Program"



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q_1	q_1	q_2
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