CS 622

Nondeterminism

Wednesday, February 14, 2024

UMass Boston Computer Science

Announcements

- HW 2 out
  - ~~Due Mon 2/19 12pm EST (noon)~~
  - Due Wed 2/21 12pm EST (noon)

Previously

Is Union Closed For Regular Langs?

In this course, **we are interested in closed operations for a** **<u>set of languages</u>** (here the **set of regular languages**)

(*In general, a* **set** is **closed** under an operation if **applying the operation** to **members of the set** produces a **result in the same set**)

The class of regular languages is closed under the union operation.

Want to prove this statement

In other words, if A_1 and A_2 are regular languages, so is $A_1 \cup A_2$.

Or this (same) **statement**

Previously

Is Union Closed For Regular Langs?

(*In general*, a **set** is **closed** under an operation if applying the operation to members of the set produces a result in the same set)

THEOREM

The class of regular languages is closed under the union operation.

In other words, if A_1 and A_2 are regular languages, so is $A_1 \cup A_2$.

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Or this (same) statement

A **member of the set of regular languages** is …

… **a regular language,** which itself is a set (of strings) …

… so the **operations** we're interested in are **set operations**

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THEOREM

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Or this (same) statement

Flashback: Mathematical Statements: IF-THEN

Using:

- If we know: $P \rightarrow Q$ is TRUE,
  what do we know about P and Q individually?
  - Either P is FALSE (not too useful, **can't prove anything about** Q), or
  - If P is TRUE, then Q is TRUE (**modus ponens**)

p	q	$p \rightarrow q$
True	True	True
True	False	False
False	True	True
False	False	True

Proving:

- To prove: $P \rightarrow Q$ is TRUE:
  - Prove P is FALSE (**usually hard** or **impossible**)
  - Assume P is TRUE, then prove Q is TRUE

Is Union Closed For Regular Langs?

Do we know anything about A_1 and A_2?

Statements	**Justifications**
1. A_1 and A_2 are regular languages	1. Assumption
2. A DFA $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ recognizes A_1	2. Def of Regular Language
3. A DFA $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ recognizes A_2	3. Def of Regular Language
4. Construct DFA $M = (Q, \Sigma, \delta, q_0, F)$ (todo)	4. Def of DFA
5. M recognizes $A_1 \cup A_2$	5. See examples
6. $A_1 \cup A_2$ is a regular language	6. Def of Regular Language
7. The class of regular languages is closed under the union operation.	7. From stmt #1 and #6

???

How to create this? Don't know what A_1 and A_2 are!

In other words, if A_1 and A_2 are regular languages, so is $A_1 \cup A_2$.

To prove $P \rightarrow Q$ is TRUE: Assume P is TRUE, then prove Q is TRUE

Wait! If A Then B =?= If B Then A

1. A_1 and A_2 are regular languages
2. A DFA $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ recognizes A_1
3. A DFA $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ recognizes A_2

1. Assumption
2. Def of Regular Language
3. Def of Regular Language

If a **DFA** recognizes a language L, then L is a **regular language**

==

If L is a **regular language**, then a **DFA** recognizes L ???

Equivalence of Conditional Statements

- Yes or No? “If X then Y” is equivalent to:
  - “If Y then X” (**converse**)
    - No!

If Regular, Then DFA?

If a **DFA recognizes** a language L, then L is a **regular language**

- Prove: If L is a **regular language**, then a **DFA recognizes** L
- Proof (Sketch)

  Case analysis:
  - **Look at all** If-then **statements** of the form:
    - "If … language L, then L is a **regular language**"
  - (At least one is true, because we know "L is a **regular language**"!)
  - **Figure out which one(s)** led to conclusion:
    - "L is a **regular language**"
  - (There's only 1!)

  - So it must be that:

"Corollary"

If L is a **regular language**, then a **DFA recognizes** L

Is Union Closed For Regular Langs?

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1. A_1 and A_2 are regular languages	1. Assumption
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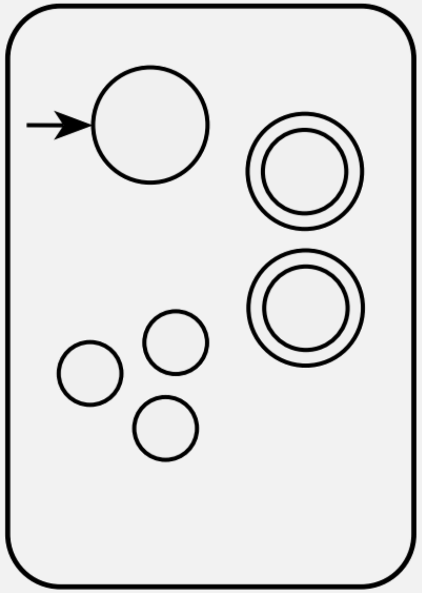
"Corollary"

???

How to create this? Don't know what A_1 and A_2 are!

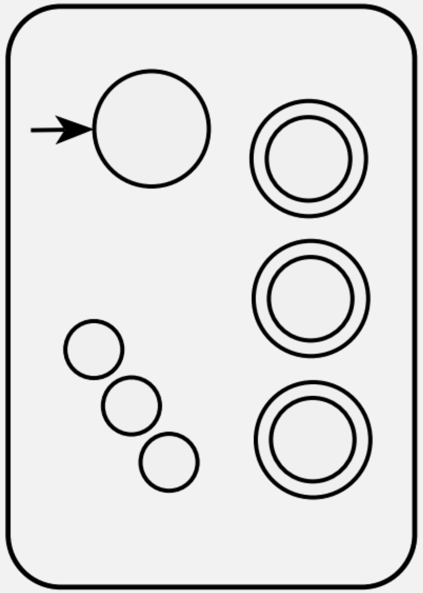
M_1

recognizes A_1



M_2

recognizes A_2



DEFINITION

A ***finite automaton*** is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

1. Q is a finite set called the ***states***,
2. Σ is a finite set called the ***alphabet***,
3. $\delta : Q \times \Sigma \longrightarrow Q$ is the ***transition function***,
4. $q_0 \in Q$ is the ***start state***, and
5. $F \subseteq Q$ is the ***set of accept states***.

Regular language A_1
Regular language A_2

Even if we <u>don't know</u> what these languages are, <u>we **still know**</u>...

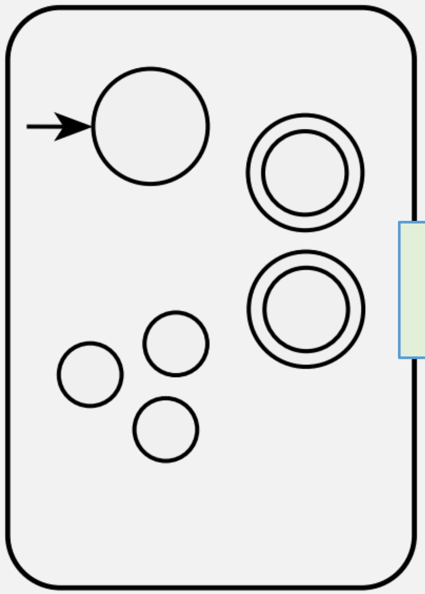
$$M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1), \text{ recognize } A_1,$$
$$M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2), \text{ recognize } A_2,$$

If L is a **regular language**, then a **DFA** recognizes L

Union

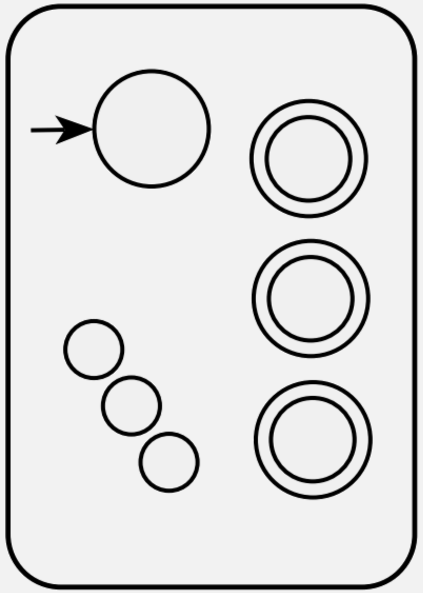
M_1

recognizes A_1



M_2

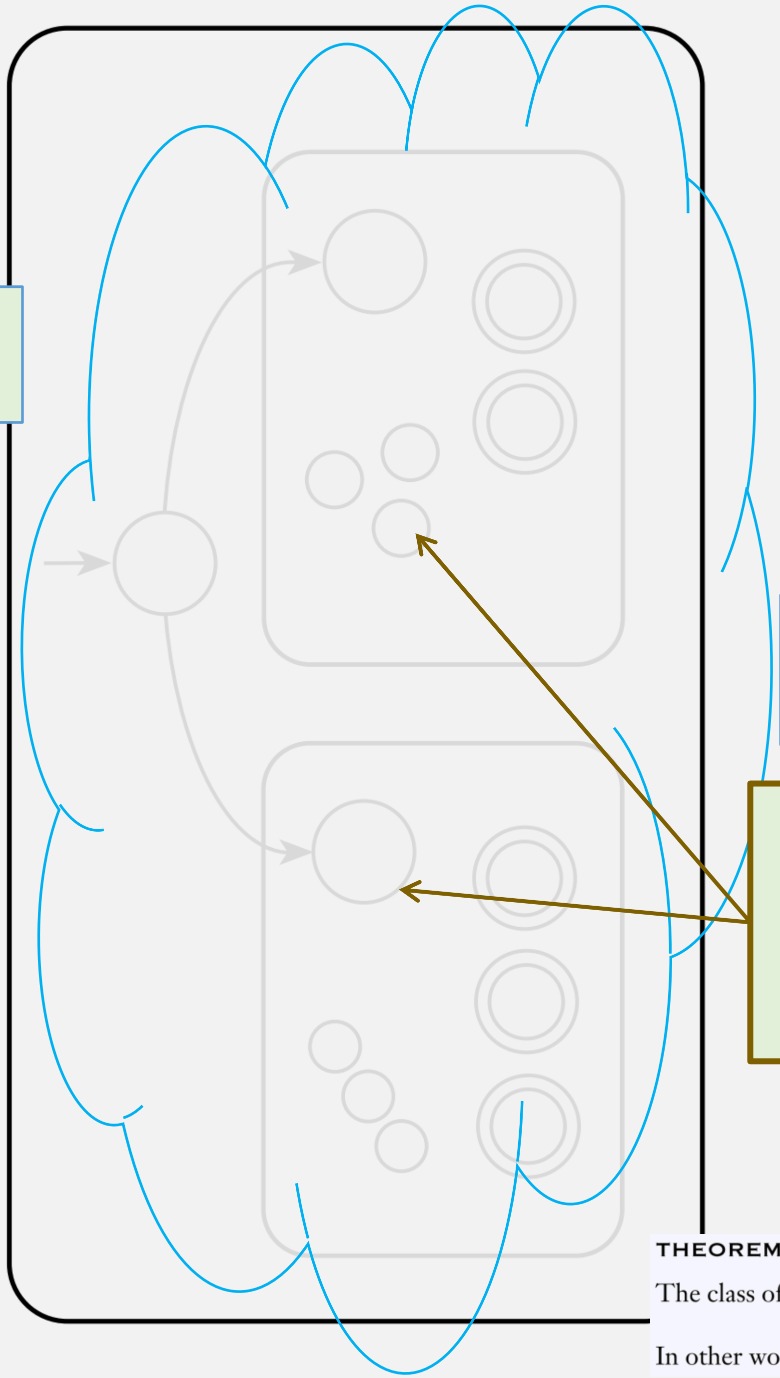
recognizes A_2



Want: M

Recognizes $A_1 \cup A_2$

(to prove $A_1 \cup A_2$ is regular)



Rough sketch Idea:
M is a combination of M_1 and M_2 that: checks whether its input is accepted by either M_1 or M_2

But, a DFA can only read its input once!

Need to somehow simulate "being in" both an M_1 and M_2 state simultaneously

THEOREM

The class of regular languages is closed under the union operation.

In other words, if A_1 and A_2 are regular languages, so is $A_1 \cup A_2$.

Union is Closed For Regular Languages

Proof (continuation)

- Given: $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$, recognize A_1,
  $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$, recognize A_2,

Want: M that can simultaneously "be in" both an M_1 and M_2 state

- Construct: $M = (Q, \Sigma, \delta, q_0, F)$, using M_1 and M_2, that recognizes $A_1 \cup A_2$

- **states of *M*:** $Q = \{(r_1, r_2) |\ r_1 \in Q_1 \text{ and } r_2 \in Q_2\}$ $= Q_1 \times Q_2$

  This set is the ***Cartesian product*** of sets Q_1 and Q_2

A ***finite automaton*** is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

1. Q is a finite set called the ***states***,
2. Σ is a finite set called the ***alphabet***,
3. $\delta : Q \times \Sigma \longrightarrow Q$ is the ***transition function***,[1]
4. $q_0 \in Q$ is the ***start state***, and
5. $F \subseteq Q$ is the ***set of accept states***.

A state of M is a pair:
- first part: state of M_1
- second part: state of M_2

states of M: all possible pair combinations of states of M_1 and M_2

Union is Closed For Regular Languages

Proof (continuation)

- Given: $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$, recognize A_1,
  $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$, recognize A_2,
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- states of M: $Q = \{(r_1, r_2) | \, r_1 \in Q_1 \text{ and } r_2 \in Q_2\}$ $= Q_1 \times Q_2$
  This set is the ***Cartesian product*** of sets Q_1 and Q_2

$\ldots a) = (\delta_1(r_1, a), \delta_2(r_2, a))$

A *finite automaton* is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

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A step in M is both:
- a step in M_1, and
- a step in M_2

Union is Closed For Regular Languages

Proof (continuation)

- Given: $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$, recognize A_1,
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  This set is the ***Cartesian product*** of sets Q_1 and Q_2
- M transition fn: $\delta\big((r_1, r_2), a\big) = \big(\delta_1(r_1, a), \delta_2(r_2, a)\big)$
- M start state: (q_1, q_2)

Start state of M is both
start states of M_1 and M_2

Union is Closed For Regular Languages

Proof (continuation)

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- M start state: (q_1, q_2)

Remember:
Accept states must be subset of Q

Accept if either M_1 or M_2 accept

- M accept states: $F = \{(r_1, r_2) | \, r_1 \in F_1 \text{ or } r_2 \in F_2\}$

Q.E.D.? ■

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Statements	Justifications
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How to create this? Don't know what A_1 and A_2 are!

“Prove” that DFA recognizes a language

Let $s_1 \in A_1$ and $s_2 \in A_2$
Let $s_3 \notin A_1$ and $s_4 \notin A_2$

Be careful when choosing examples!

In this class, **a table like this is sufficient to “prove” that a DFA recognizes a language**

String	In lang $A_1 \cup A_2$?	Accepted by M ?
s_1	Yes	
s_2	Yes	
s_3	???	
s_4	???	

Don’t know A_1 and A_2 exactly … … but we know … … they are **sets of strings**!

$M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$, recognize A_1,
$M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$, recognize A_2,

constructed $M = (Q, \Sigma, \delta, q_0, F)$ recognizes $A_1 \cup A_2$?

“Prove” that DFA recognizes a language

Let $s_1 \in A_1$ and $s_2 \in A_2$

~~Let $s_3 \notin A_1$ and $s_4 \notin A_2$~~

Let $s_5 \notin A_1$ and $\notin A_2$

String	In lang $A_1 \cup A_2$?	Accepted by M ?
s_1	Yes	
s_2	Yes	
~~s_3~~	~~???~~	
~~s_4~~	~~???~~	
s_5	No	

$M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$, recognize A_1,
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constructed $M = (Q, \Sigma, \delta, q_0, F)$ recognizes $A_1 \cup A_2$?

Union is Closed For Regular Languages

Proof (continuation)

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- M start state: (q_1, q_2)

Accept if <u>either</u> M_1 or M_2 accept

- M accept states: $F = \{(r_1, r_2) \mid r_1 \in F_1 \text{ or } r_2 \in F_2\}$

"Prove" that DFA recognizes a language

Let $s_1 \in A_1$ and $s_2 \in A_2$

Let $s_5 \notin A_1$ and $\notin A_2$

String	In lang $A_1 \cup A_2$?	Accepted by M ?
s_1	Yes	Accept
s_2	Yes	Accept
~~s_3~~	~~???~~	???
~~s_4~~	~~???~~	???
s_5	No	Reject

$M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$, recognize A_1,
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constructed $M = (Q, \Sigma, \delta, q_0, F)$ Accept if <u>either</u> M_1 or M_2 accept

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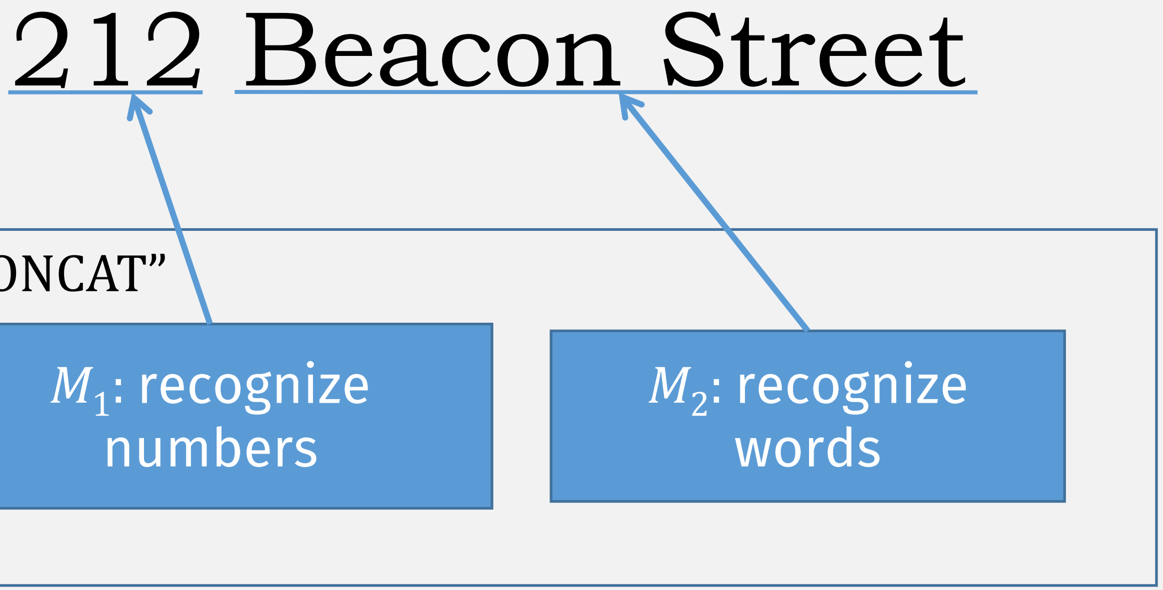
Q.E.D. ■

Another operation: Concatenation

Example: **Recognizing street addresses**



Concatenation: $A \circ B = \{xy | \, x \in A \text{ and } y \in B\}$

Concatenation of Languages

Let the alphabet Σ be the standard 26 letters $\{\texttt{a}, \texttt{b}, \ldots, \texttt{z}\}$.

If $A = \{\texttt{fort}, \texttt{south}\}$ $B = \{\texttt{point}, \texttt{boston}\}$

$$A \circ B = \{\texttt{fortpoint}, \texttt{fortboston}, \texttt{southpoint}, \texttt{southboston}\}$$

Is Concatenation Closed?

THEOREM

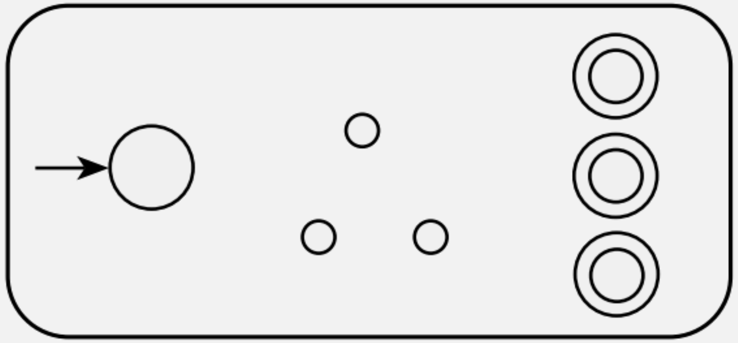
The class of regular languages is closed under the concatenation operation.

In other words, if A_1 and A_2 are regular languages then so is $A_1 \circ A_2$.

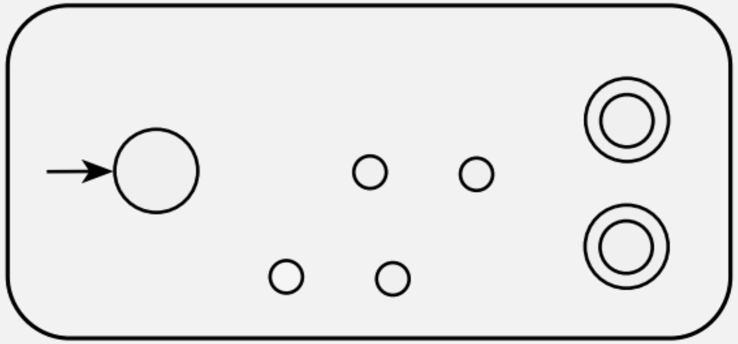
- **Construct a <u>new</u> machine M recognizing $A_1 \circ A_2$?** (like union)
  - Using **DFA M_1** (which recognizes A_1),
  - and **DFA M_2** (which recognizes A_2)

Concatentation

M_1



M_2

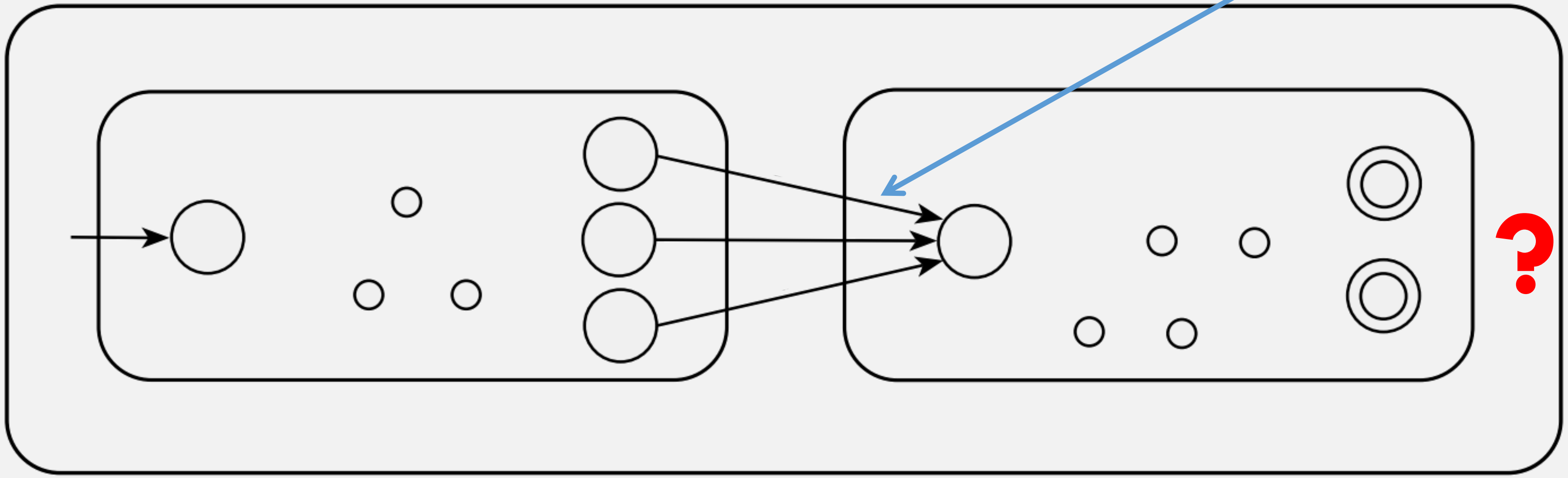


PROBLEM: Can only read input once, can't backtrack

Let M_1 recognize A_1, and M_2 recognize A_2.

Want: Construction of M to recognize $A_1 \circ A_2$

Need to switch machines at some point, but when?



???

Concatenation: $A \circ B = \{xy | x \in A \text{ and } y \in B\}$

Overlapping Concatenation Example

- Let M_1 recognize language A = { `jen`, `jens` }
- and M_2 recognize language B = { `smith` }
- Want: Construct M to recognize $A \circ B$ = { `jensmith`, `jenssmith` }

- If M sees `jen` …
- M must decide to either:

Concatenation: $A \circ B = \{xy \mid x \in A \text{ and } y \in B\}$

Overlapping Concatenation Example

- Let M_1 recognize language A = { `jen`, `jens` }
- and M_2 recognize language B = { `smith` }
- Want: Construct M to recognize $A \circ B$ = { `jensmith`, `jenssmith` }

- If M sees `jen` ...
- M must decide to either:
  - stay in M_1 (correct, if full input is `jenssmith`)

Concatenation: $A \circ B = \{xy|\ x \in A \text{ and } y \in B\}$

Overlapping Concatenation Example

- Let M_1 **recognize language** A = { `jen`, `jens` }
- and M_2 **recognize language** B = { `smith` }
- Want: **Construct** M **to recognize** $A \circ B$ = { `jensmith`, `jenssmith` }

- If M **sees** `jen` ...
- M **must decide to** **either**:
  - **stay in** M_1 (correct, if full input is `jenssmith`)
  - or **switch to** M_2 (correct, if full input is `jensmith`)
- **But to recognize** $A \circ B$**, it needs to handle both cases!!**
  - **Without backtracking**

A DFA can't do this!

Is Concatenation Closed?

FALSE?

THEOREM

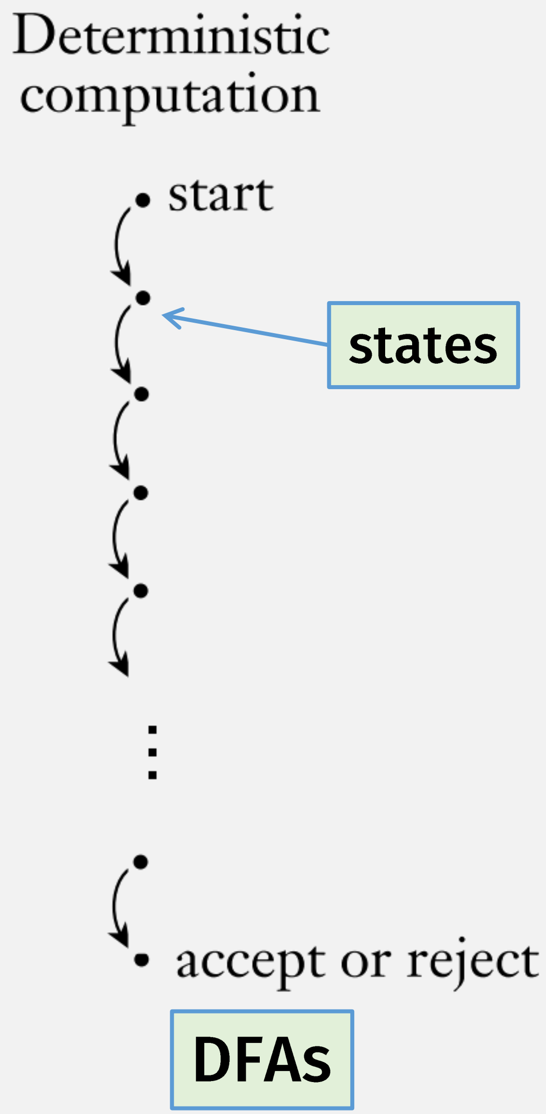
The class of regular languages is closed under the concatenation operation.

In other words, if A_1 and A_2 are regular languages then so is $A_1 \circ A_2$.

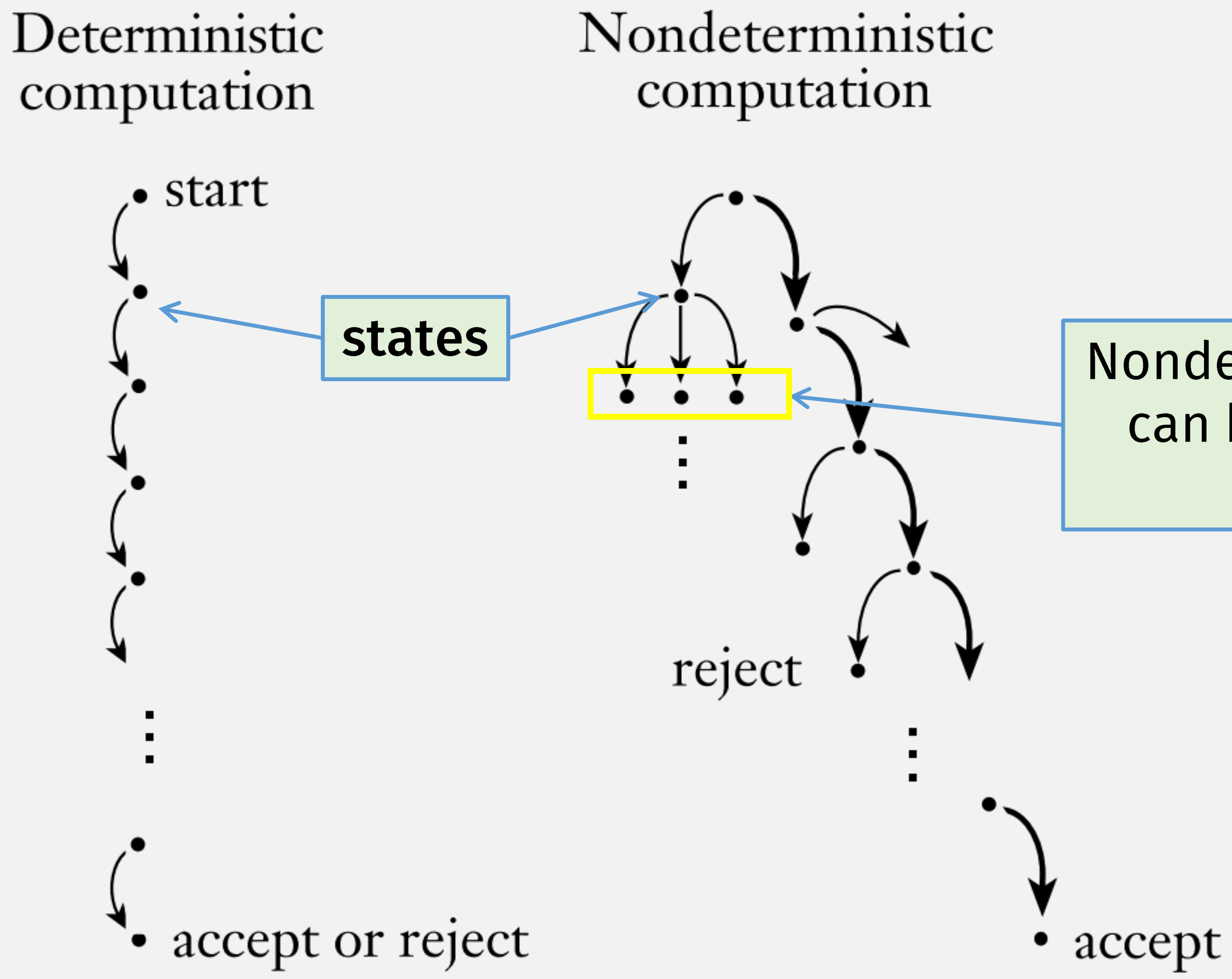
- Cannot **combine A_1 and A_2's machine because:**
  - **Need to switch from A_1 to A_2 at some point …**
  - **… but we don't know when!** (we can only read input once)
- **This requires a new kind of machine!**
- But **does this mean concatenation is not closed for regular langs?**

Nondeterminism

Deterministic vs Nondeterministic



Deterministic vs Nondeterministic



Nondeterministic computation can be in multiple states at the same time

DFAs

New FA

Previously

DFAs: The Formal Definition

DEFINITION

deterministic

A ***finite automaton*** is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

1. Q is a finite set called the ***states***,
2. Σ is a finite set called the ***alphabet***,
3. $\delta : Q \times \Sigma \longrightarrow Q$ is the ***transition function***,
4. $q_0 \in Q$ is the ***start state***, and
5. $F \subseteq Q$ is the ***set of accept states***.

Deterministic Finite Automata (DFA)

Nondeterministic Finite Automata (NFA)

DEFINITION

A ***nondeterministic finite automaton*** is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

1. Q is a finite set of states,
2. Σ is a finite alphabet,
3. $\delta \colon Q \times \Sigma_\varepsilon \longrightarrow \mathcal{P}(Q)$ is the transition function,
4. $q_0 \in Q$ is the start state, and
5. $F \subseteq Q$ is the set of accept states.

Compare with DFA:

A ***finite automaton*** is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

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5. $F \subseteq Q$ is the ***set of accept states***.

Difference

Power set, i.e. a transition results in set of states

Power Sets

- A **power set** is the set of all subsets of a set

- Example: S = {**a**, **b**, **c**}

- Power set of S =
  - { { }, {**a**}, {**b**}, {**c**}, {**a**, **b**}, {**a**, **c**}, {**b**, **c**}, {**a**, **b**, **c**} }
  - Note: includes the empty set!

Nondeterministic Finite Automata (NFA)

DEFINITION

A ***nondeterministic finite automaton*** is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

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4. $q_0 \in Q$ is the start state, and
5. $F \subseteq Q$ is the set of accept states.

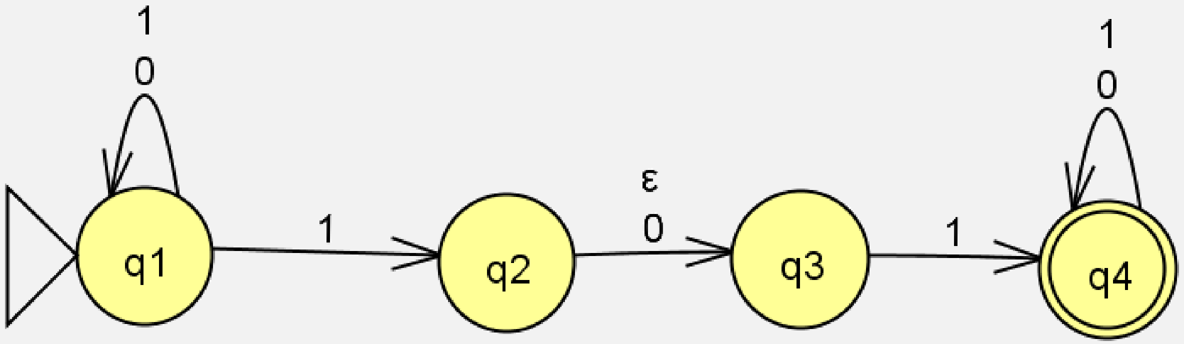
Transition label can be "empty", i.e., machine can transition without reading input

$$\Sigma_\varepsilon = \Sigma \cup \{\varepsilon\}$$

CAREFUL:
ε symbol is reused here, as a transition label.
- It's not the empty string!
- And it's (still) not a character in the alphabet Σ!

NFA Example

- Come up with a formal description of the following NFA:



DEFINITION

A ***nondeterministic finite automaton*** is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

1. Q is a finite set of states,
2. Σ is a finite alphabet,
3. $\delta\colon Q \times \Sigma_\varepsilon \longrightarrow \mathcal{P}(Q)$ is the transition function,
4. $q_0 \in Q$ is the start state, and
5. $F \subseteq Q$ is the set of accept states.

The formal description of N_1 is $(Q, \Sigma, \delta, q_1, F)$, where

$$\delta : Q \times \Sigma_\varepsilon \longrightarrow \mathcal{P}(Q)$$

1. $Q = \{q_1, q_2, q_3, q_4\}$,
2. $\Sigma = \{0,1\}$,
3. δ is given as

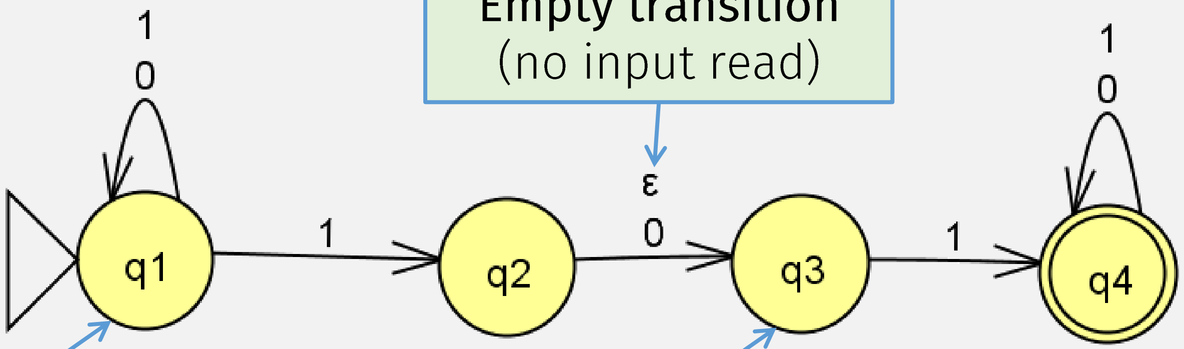
Empty transition (no input read)

	0	1	ε
q_1	$\{q_1\}$	$\{q_1, q_2\}$	$\emptyset$
q_2	$\{q_3\}$	$\emptyset$	$\{q_3\}$
q_3	$\emptyset$	$\{q_4\}$	$\emptyset$
q_4	$\{q_4\}$	$\{q_4\}$	$\emptyset$,

Result of transition is a set

4. q_1 is the start state, and
5. $F = \{q_4\}$.

Empty transition (no input read)

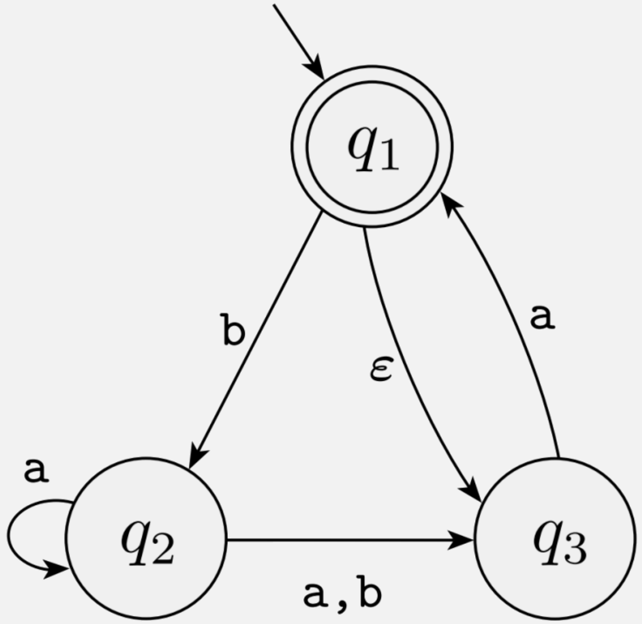


Multiple 1 transitions

No 0 transition

In-class Exercise

- Come up with a formal description for the following NFA
  - $\Sigma = \{ \texttt{a}, \texttt{b} \}$



DEFINITION

A ***nondeterministic finite automaton*** is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

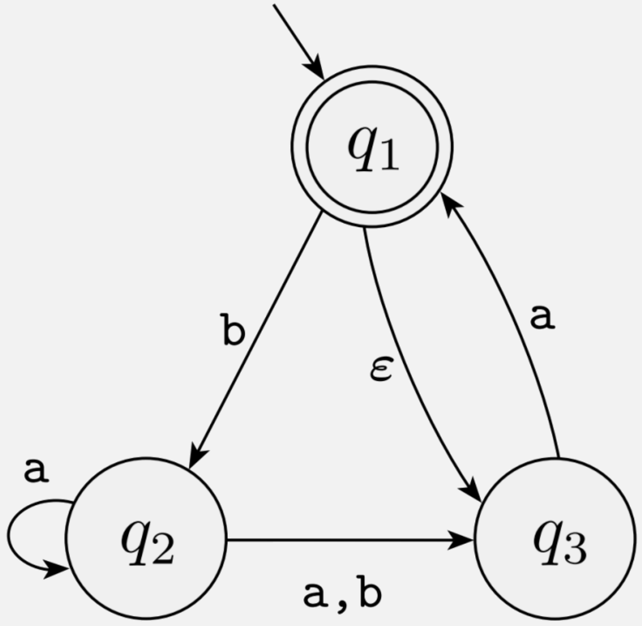
1. Q is a finite set of states,
2. Σ is a finite alphabet,
3. $\delta \colon Q \times \Sigma_\varepsilon \longrightarrow \mathcal{P}(Q)$ is the transition function,
4. $q_0 \in Q$ is the start state, and
5. $F \subseteq Q$ is the set of accept states.

In-class Exercise Solution

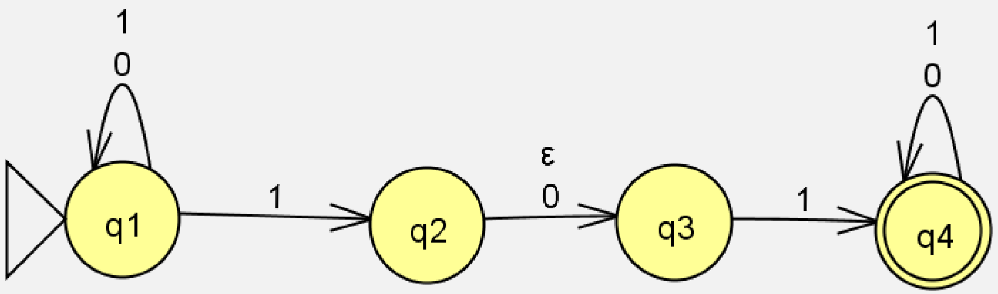
Let $N = (Q, \Sigma, \delta, q_0, F)$

- $Q = \{ q_1, q_2, q_3 \}$
- $\Sigma = \{ \texttt{a}, \texttt{b} \}$
- δ ...
- $q_0 = q_1$
- $F = \{ q_1 \}$

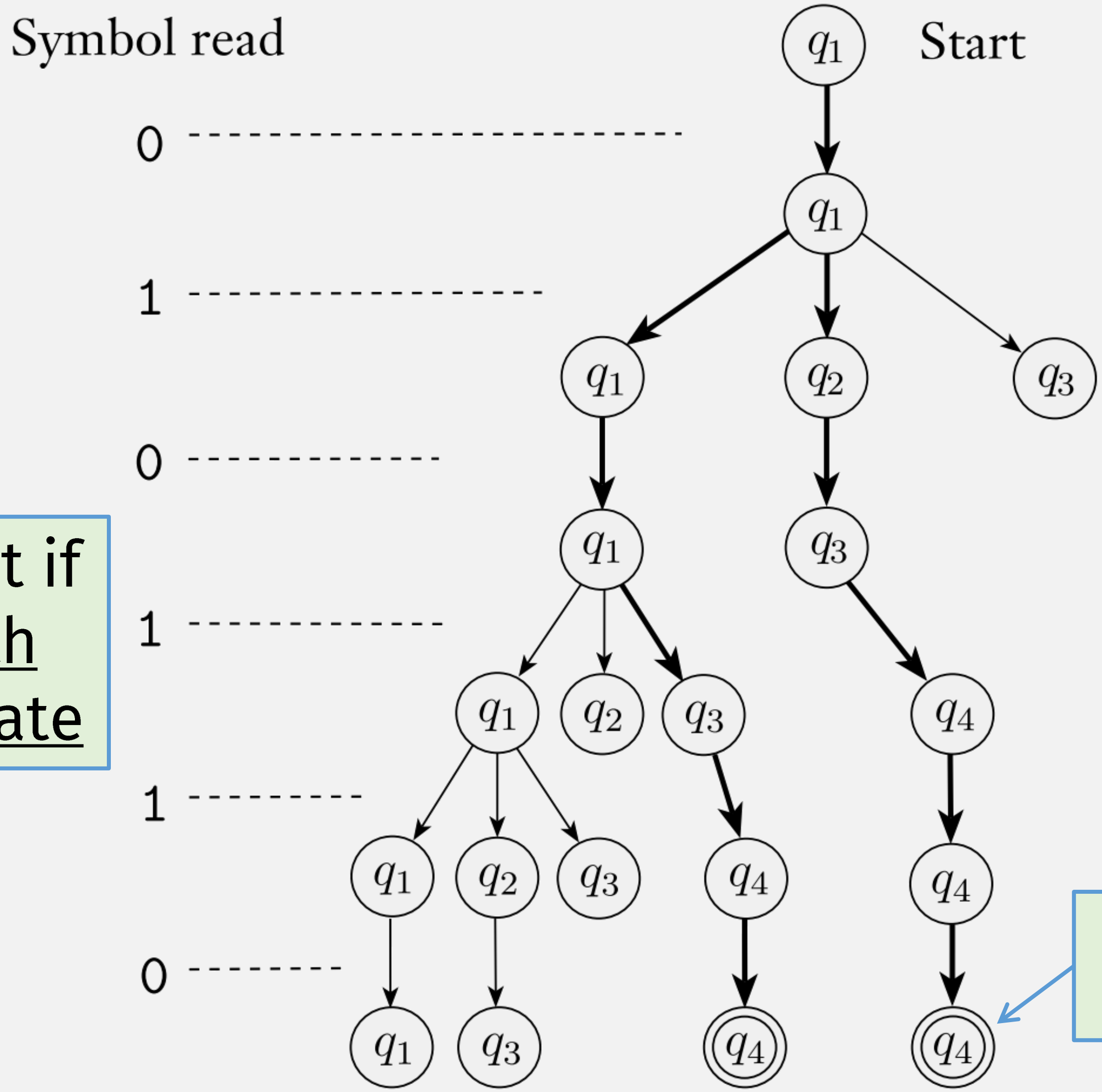
$\delta(q_1, \texttt{a}) = \{ \}$
$\delta(q_1, \texttt{b}) = \{ q_2 \}$
$\delta(q_1, \varepsilon) = \{ q_3 \}$
$\delta(q_2, \texttt{a}) = \{ q_2, q_3 \}$
$\delta(q_2, \texttt{b}) = \{ q_3 \}$
$\delta(q_2, \varepsilon) = \{ \}$
$\delta(q_3, \texttt{a}) = \{ q_1 \}$
$\delta(q_3, \texttt{b}) = \{ \}$
$\delta(q_3, \varepsilon) = \{ \}$



NFA Computation (JFLAP demo): `010110`



NFA Computation Sequence



NFA **accepts** input if at least one path ends in accept state

Each step can branch into multiple states at the same time!

So this is an **accepting computation**

Submit in-class work 2/14

On gradescope