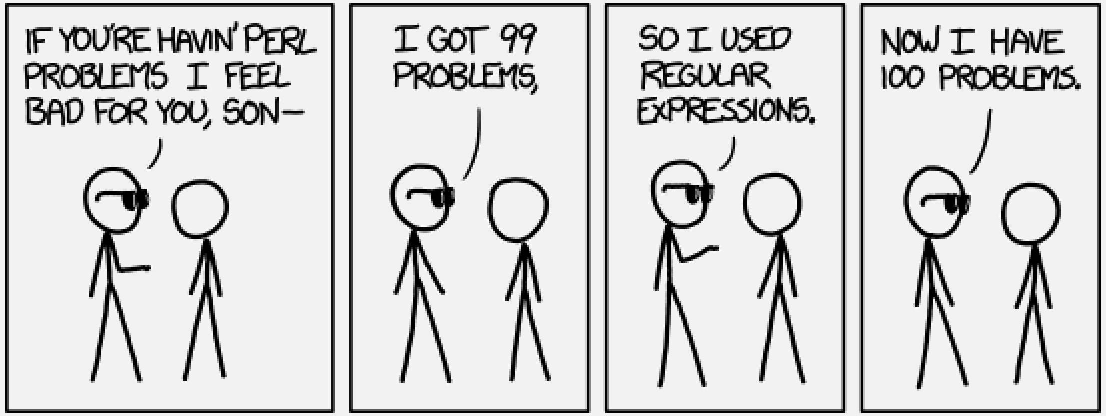
CS420
Finite Automata and Regular Languages

Instructor: Stephen Chang

Mon Sept 14, 2020

UMass Boston Computer Science

HW 0 Questions?



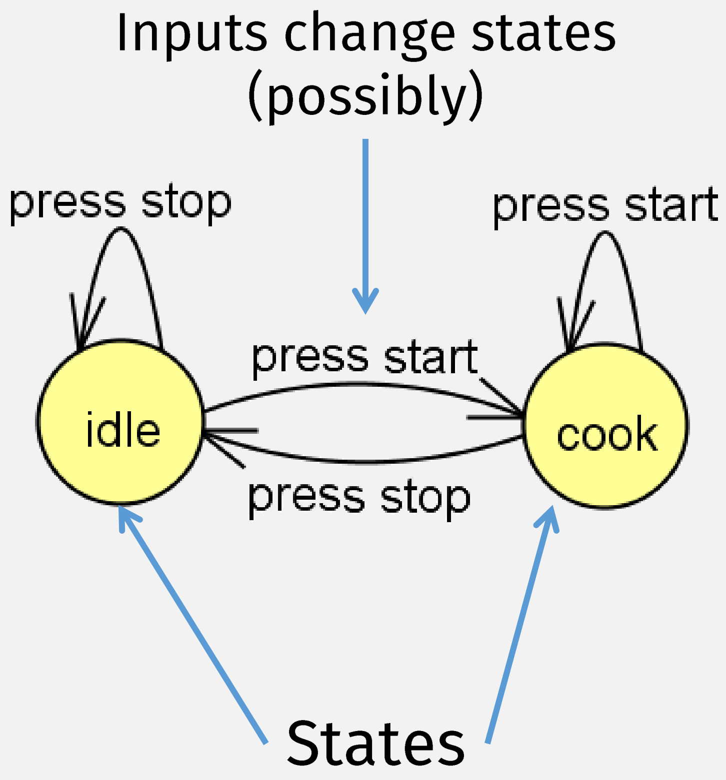
Quick Poll: Regular Expressions

The Good, the Bad, the … Weird?

Deterministic Finite Automata (DFAs)

A computational model for …

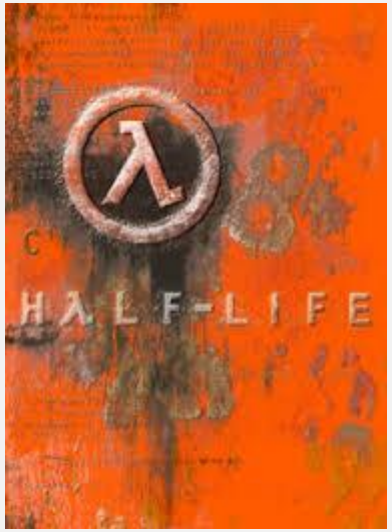
A Finite Automata (or State Machine)



Finite Automata: Not Just for Microwaves

Finite Automata:
a common
programming pattern

Finite Automata in: Video Games



ValveSoftware / **halflife**

Code | Issues 1.6k | Pull requests 23 | Actions | Projects | Wiki

5d761709a3 ▾ halflife / game_shared / bot / **simple_state_machine.h**

Alfred Reynolds initial seed of Half-Life 1 SDK

0 contributors

85 lines (67 sloc) | 2.15 KB

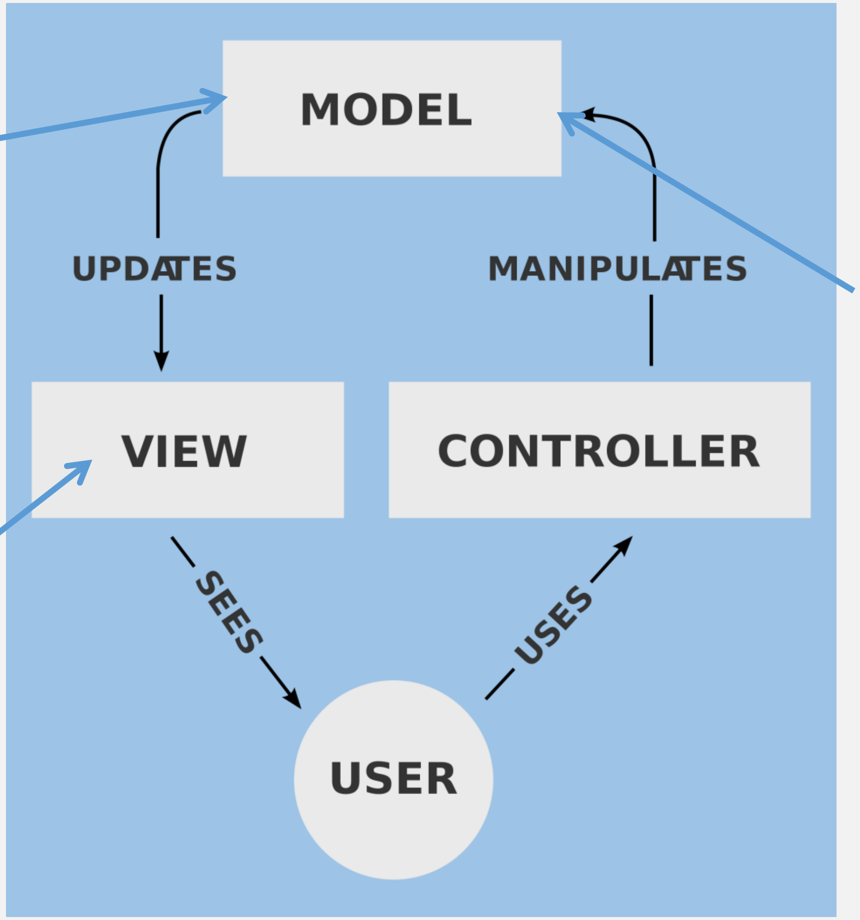
```
// simple_state_machine.h
// Simple finite state machine encapsulation
// Author: Michael S. Booth (mike@turtlerockstudios.com), November 2003

#ifndef _SIMPLE_STATE_MACHINE_H_
#define _SIMPLE_STATE_MACHINE_H_

//----------------------------------------------------------------------------------------
/**
 * Encapsulation of a finite-state-machine state
 */
template < typename T >
class SimpleState
```

Model-view-controller (MVC) is a FSM

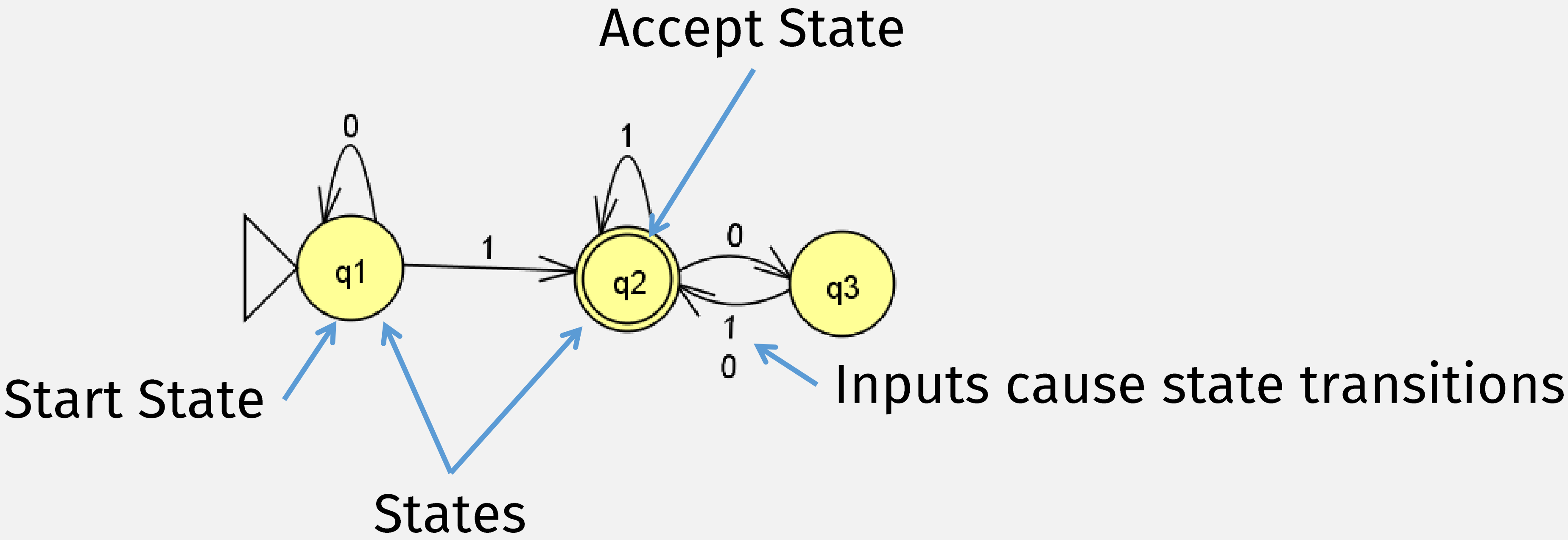
States



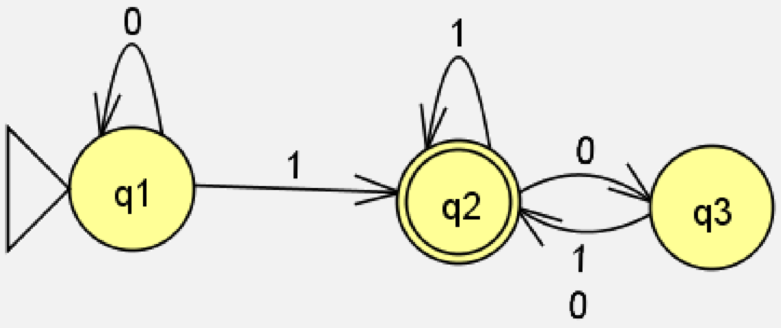
Inputs change states

View Draw states

Finite Automata in this class: state diagram



JFLAP demo: “Running” an FSM “Program”

- FSM: 

- Program: “1101”

Finite Automata: The Formal Definition

DEFINITION 1.5

A ***finite automaton*** is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

1. Q is a finite set called the ***states***,
2. Σ is a finite set called the ***alphabet***,
3. $\delta : Q \times \Sigma \longrightarrow Q$ is the ***transition function***,
4. $q_0 \in Q$ is the ***start state***, and
5. $F \subseteq Q$ is the ***set of accept states***.

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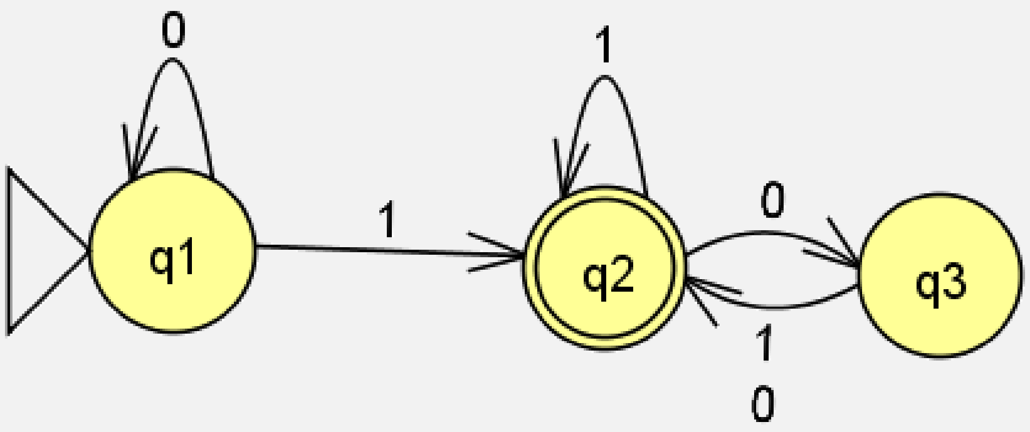

$M_1 = (Q, \Sigma, \delta, q_1, F)$, where

1. $Q = \{q_1, q_2, q_3\}$,
2. $\Sigma = \{0,1\}$,
3. δ is described as

	0	1
q_1	q_1	q_2
q_2	q_3	q_2
q_3	q_2	q_2,

4. q_1 is the start state, and
5. $F = \{q_2\}$.

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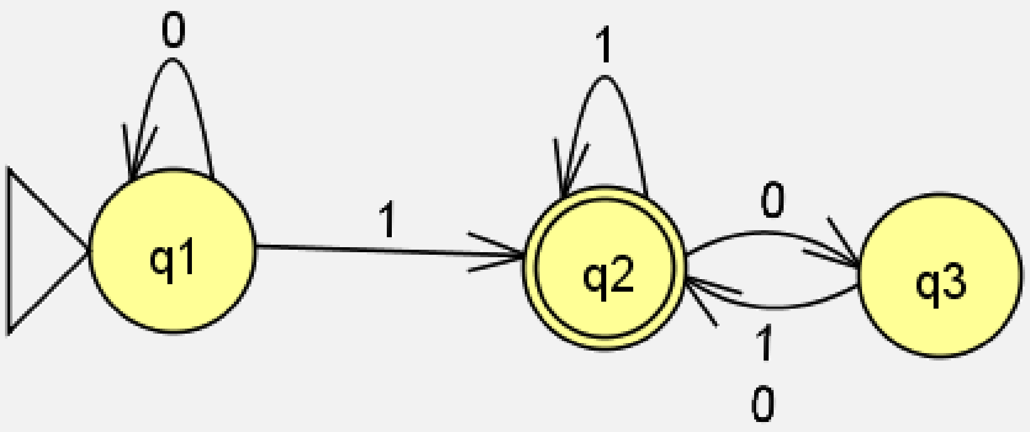

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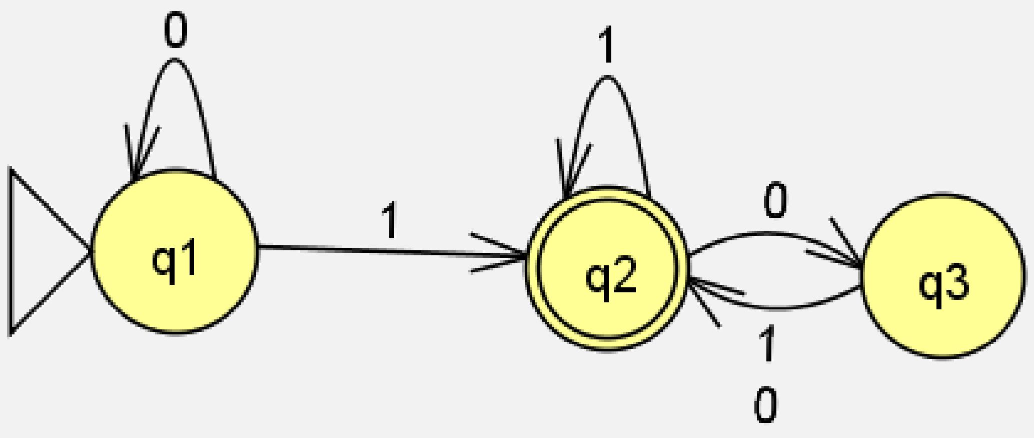

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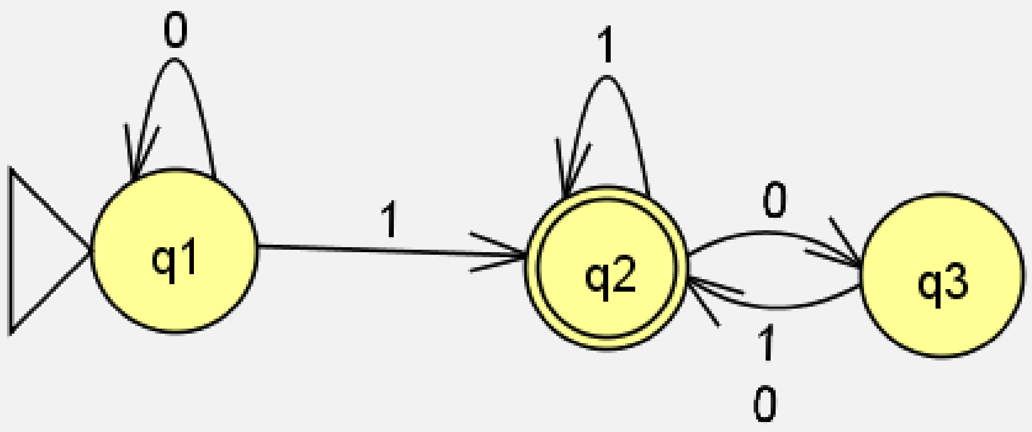

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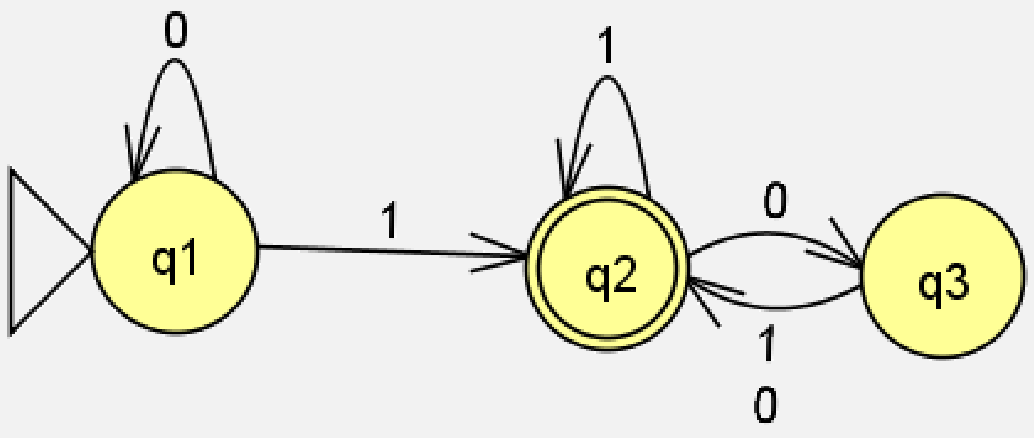

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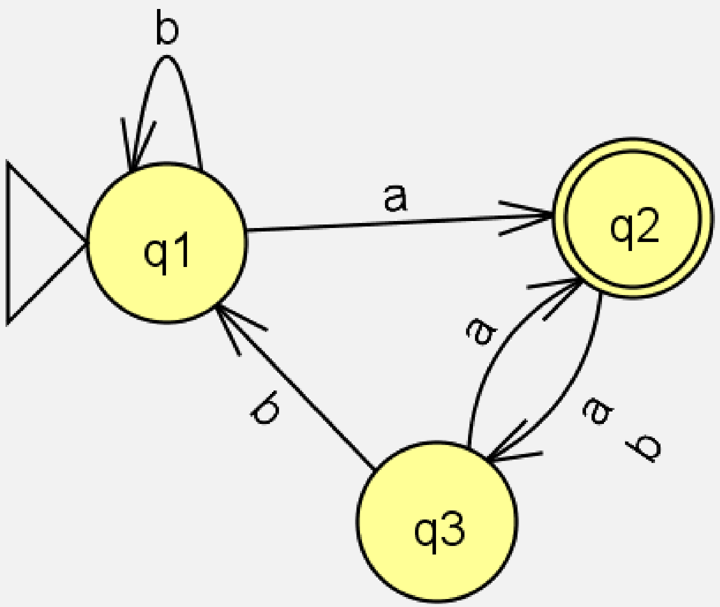
In-class exercise

- Come up with a formal description of the following machine:



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Terminology

- These are all equivalent:
  - Finite State Machine (FSM)
  - Finite Automaton, Automata, Automaton
  - State Machine
- They <u>generally</u> describe the class of machine**s** studied in Ch 1

- What I just introduced:
  - Deterministic Finite Automata (DFA)
- A specific kind of FSM, corresponding to Definition 1.5

- <u>**At this point** in the course</u> all terms on this slide are the same
  - But they wont be later

Math vs Its Code Representation

- In CS420 we use code to explore mathematical objects
- But it's important to understand the distinction
- E.g., a set is an <u>abstract</u> mathematical object
  - contains other math objects like: strings, nums, characters, and other sets!
- A set's (data) <u>representation</u> in code can take many forms:
  - e.g., a list, an array, a space-separated string
- This course teaches abstract mathematical concepts
  - It is up to you how to represent the math as code and data!

Math vs Representation, Examples

Abstract Math Concept	Possible Data Representation
Numbers	
Set	
Tuple (i.e., a small finite set)	
Function, i.e., a set of pairs	
Finite automata	

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Finite automata	XML str, <your choice here>

“Running a Program” on a Finite Automata

- Program = an input string of characters
- Start in “Start State”
- One char at a time, follow transition table to change states
- Result of running the program:
  - “Accept” the input if last state is an “Accept State”
  - “Reject” otherwise

Formal Definition of "Computation"

$M = (Q, \Sigma, \delta, q_0, F)$ a finite automaton
$w = w_1 w_2 \cdots w_n$ a string where each w_i is a member of the alphabet Σ.

M ***accepts*** w if a sequence of states $r_0, r_1, \ldots, r_n$ in Q exists with three conditions:

1. $r_0 = q_0$,
2. $\delta(r_i, w_{i+1}) = r_{i+1}$, for $i = 0, \ldots, n-1$, and
3. $r_n \in F$.

Condition 1 machine starts in the start state.
Condition 2 machine goes from state to state according to the transition function.
Condition 3 machine accepts its input if it ends up in an accept state.

Terminology

- M ***accepts*** w
- M ***recognizes language*** A

  if $A = \{w | \ M \text{ accepts } w\}$
- A language is called a ***regular language*** if some finite automaton recognizes it.

Proving that a language is regular

Kinds of Mathematical Proof

- Proof by construction
  - Construct the mathematical object in question
- Proof by contradiction

- Proof by induction

Proving that a language is regular

- Often requires creating a FSM

A language is called a ***regular language*** if some finite automaton recognizes it.

Designing Finite Automata

- States = the machine's **memory**!
  - Finite amount of memory: must be allocated in advance
  - Think about what information must be remembered.

- <u>Example</u>: machine accepts strings with even number of 0s
  - Two states: 1) seen even number of 0s, 2) seen odd number of 0s

- Input may only be read once

- Must decide accept/reject after that

In-class example

- Design machine M that recognizes: {w |w has exactly three 1's}

- Where Σ= {0, 1},

- Remember:

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Check-in Quiz 1

https://www.gradescope.com/courses/160337/assignments/650219

End of Class survey 9/14

https://forms.gle/pZqmX3urYRN5sn3t5