UMass Boston Computer Science

CS450 High Level Languages (section 2)

Tree Data Definitions

Wednesday, October 25, 2023

Logistics

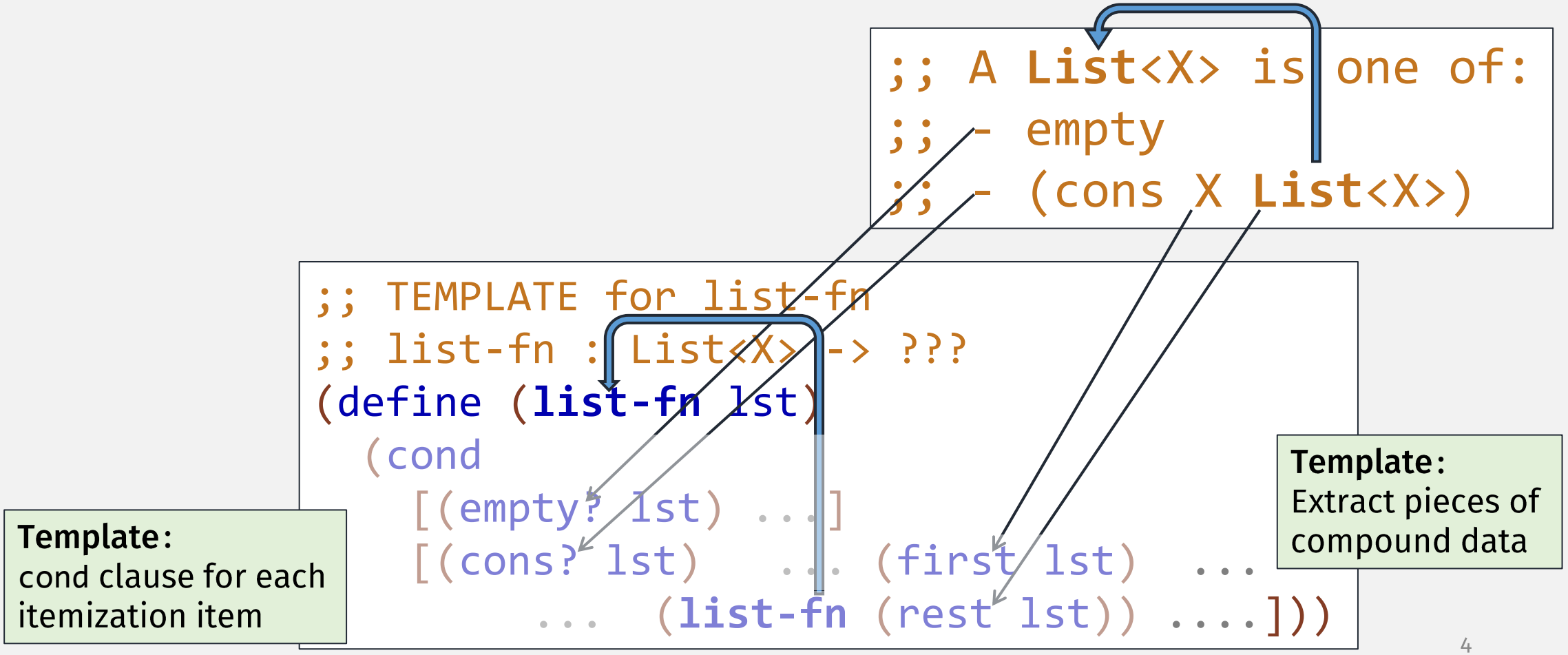
- HW 4 in
  - ~~due: Sun 10/22 11:59 pm EST~~

- HW 5 out
  - **UPDATE**: split into two parts
  - Part 1 due: Sun 10/29 11:59 pm EST
  - Part 2 due: Sun 11/5 11:59 pm EST

Previously

Recursive Data Definitions

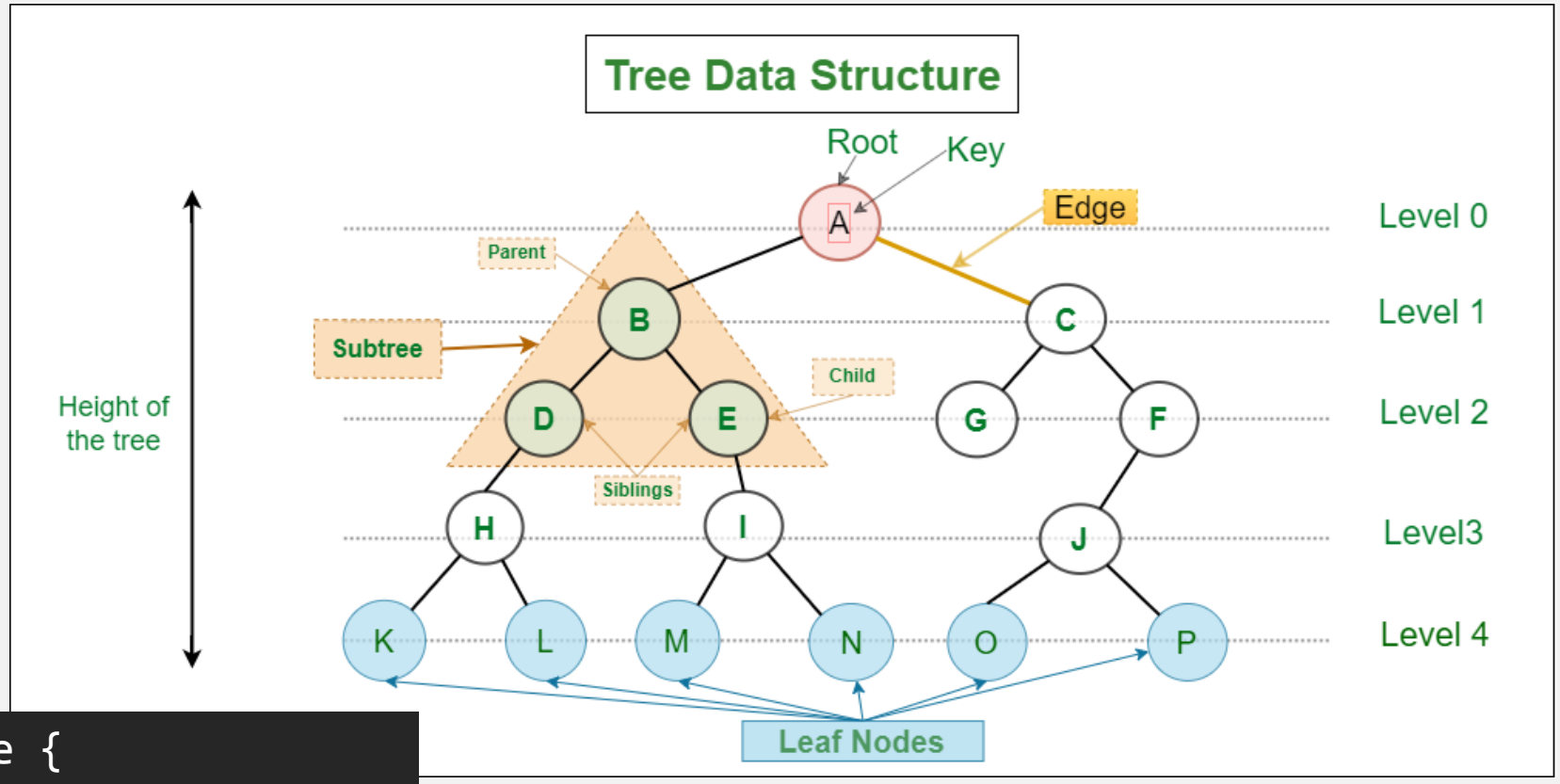
Template:
Recursive call matches recursion in data definition



Template:
cond clause for each itemization item

Template:
Extract pieces of compound data

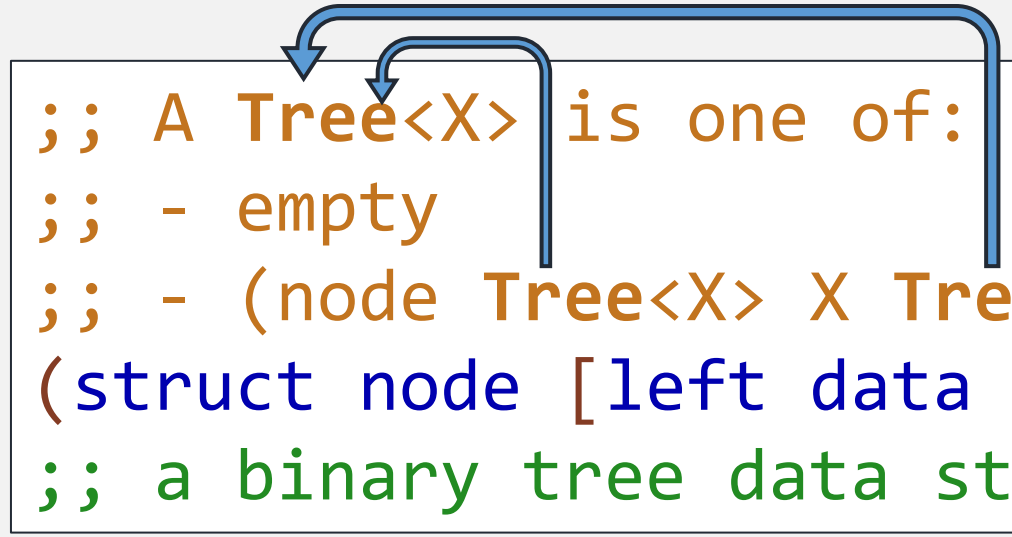
Another Data Structure: Trees




```
struct node {
        int data;
        struct node* left;
        struct node* right;
};
```

A **Tree** is a recursive data structure!

More Recursive Data Definitions: Trees



```
;; A Tree<X> is one of:
;; - empty
;; - (node Tree<X> X Tree<X>)
(struct node [left data right])
;; a binary tree data structure
```

```
;; A List<X> is one of:
;; - empty
;; - (cons X List<X>)
```

```
(define (tree? x) (or (empty? x) (node? x)))
```

(predicate only does top-level check)

```
struct node {
        int data;
        struct node* left;
        struct node* right;
};
```

More Recursive Data Definitions: Trees

```
;; A Tree<X> is one of:
;; - empty
;; - (node Tree<X> X Tree<X>)
(struct node [left data right])
;; a binary tree data structure
```

Template:
Recursive call matches recursion in data definition

Template?

Template:
Extract pieces of compound data

Template:
cond clause for each itemization item

In-class Coding

- `git clone git@github.com:cs450f23/lecture14-inclass`
- `git add tree-template-<`*your last name*`>.rkt`
  - E.g., `tree-template-chang.rkt`
- `git commit tree-template-chang.rkt –m ‘add chang tree template’`
- `git push origin main`
- Might need: `git pull --rebase`
  - If someone pushed before you, and your local clone is not at `HEAD`

(Will get quiz / participation extra credit)

In-class Coding #1: Write the Tree Template

```
;; A Tree<X> is one of:
;; - empty
;; - (node Tree<X> X Tree<X>)
(struct node [left data right])
;; a binary tree data structure
```

Template:
Recursive call matches recursion in data definition

Template:
cond clause for each itemization item

Template:
Extract pieces of compound data

- `git clone git@github.com:cs450f23/lecture14-inclass`
- `git add tree-template-`*<your last name>*`.rkt`
  - E.g., `tree-template-chang.rkt`
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- `git push origin main`
- Might need: `git pull --rebase`
  - If someone pushed before you, and your local clone is not at HEAD

In-class Coding #1: Tree Template

```
;; A Tree<X> is one of:
;; - empty
;; - (node Tree<X> X Tree<X>)
(struct node [left data right])
;; a binary tree data structure
```

```
;; tree-fn : Tree<X> -> ???
(define (tree-fn t)
  (cond
    [(empty? t) ...]
    [(node? t) ... (tree-fn (node-left t)) ...
                   ... (node-data t) ...
                   ... (tree-fn (node-right t)) ...]))
```

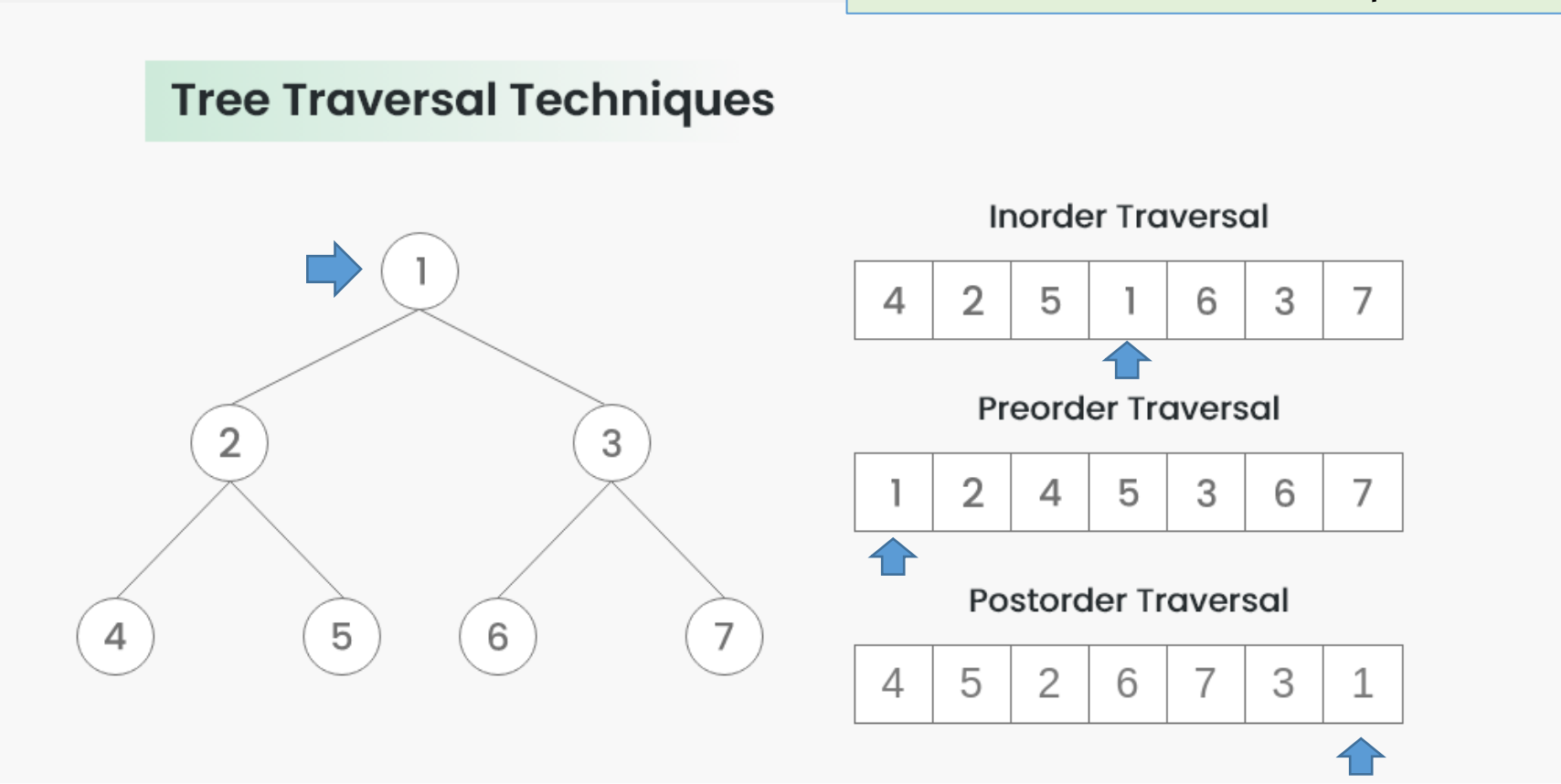
Template:
cond clause for each itemization item

Template:
Recursive call(s) match recursion in data definition

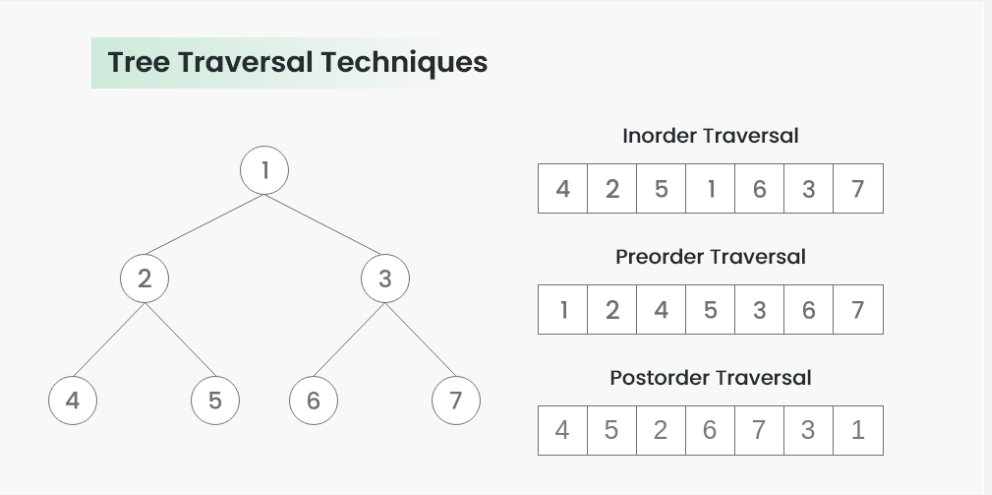
Template:
Extract pieces of compound data

Tree Algorithms

Main difference: when to process root node



Tree Algorithms



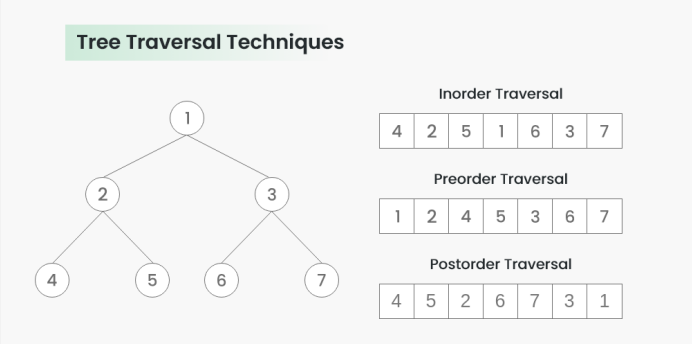

```
;; tree->lst/in : Tree<X> -> List<X>
;; converts given tree to a list of values, by inorder
```

```
;; tree->lst/pre : Tree<X> -> List<X>
;; converts given tree to a list of values, by preorder
```

```
;; tree->lst/post : Tree<X> -> List<X>
;; converts given tree to a list of values, by postorder
```

In-class Coding #2: Use the Template

```
;; A Tree<X> is one of:
;; - empty
;; - (node Tree<X> X Tree<X>)
(struct node [left data right])
;; a binary tree data structure
```




```
;; tree->lst/in : Tree<X> -> List<X>
;; converts given tree to a list of
values, by inorder
```

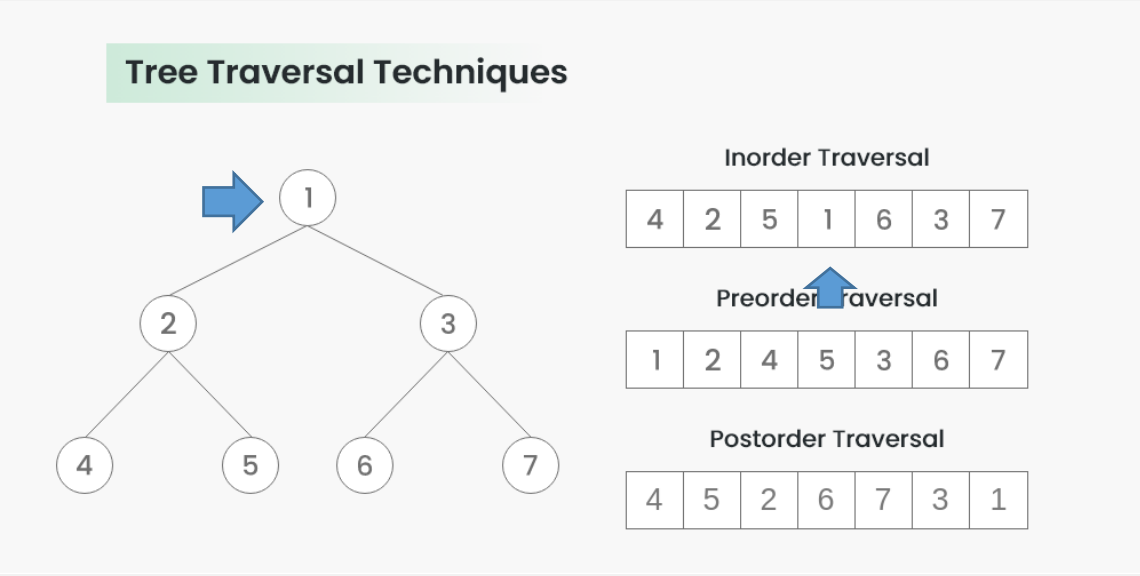
```
;; tree->lst/pre : Tree<X> -> List<X>
;; converts given tree to a list of
values, by preorder
```

```
;; tree->lst/post : Tree<X> -> List<X>
;; converts given tree to a list of
values, by postorder
```

- `git add tree-traversal-<`*your last name*`>.rkt`
  - E.g., `tree-traversal-chang.rkt`
- `git commit tree-traversal-chang.rkt -m 'add chang tree traversal'`
- `git push origin main`
- Might need: `git pull --rebase`
  - If your local clone is not at HEAD

```
;; tree-fn : Tree<X> -> ???
(define (tree-fn t)
  (cond
    [(empty? t) ...]
    [(node? t) ... (tree-fn (node-left t)) ...
                   ... (node-data t) ...
                   ... (tree-fn (node-right t)) ...]))
```

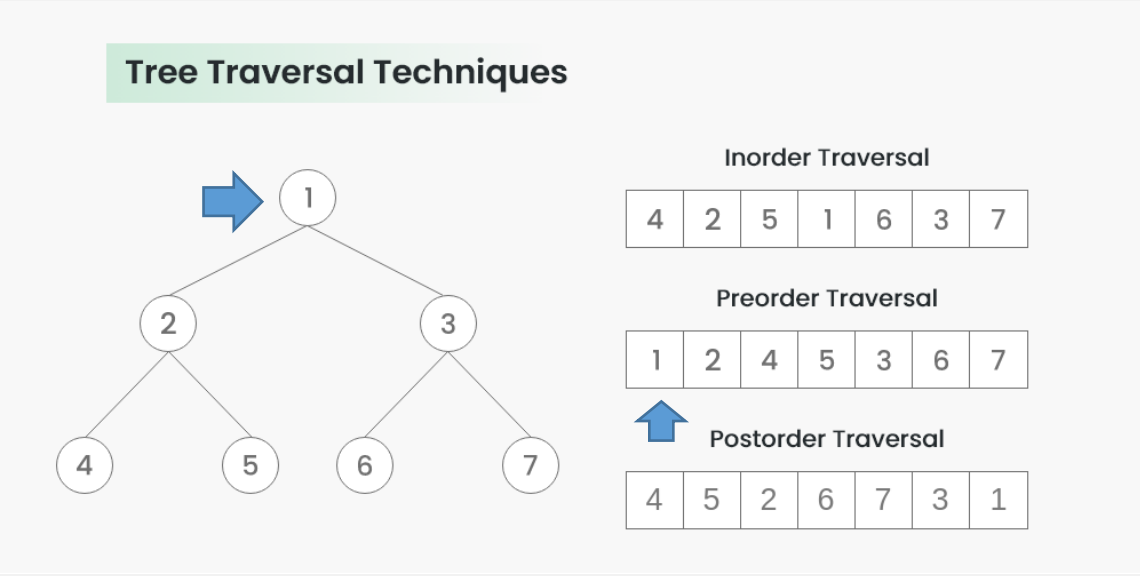
In-order Traversal




```
;; tree->lst/in : Tree<X> -> List<X>
;; converts given tree to a list of values, by inorder
```

```
(define (tree->lst/in t)
  (cond
    [(empty? t) empty]
    [(node? t) (append (tree->lst/in (node-left t))
                       (cons (node-data t)
                             (tree->lst/in (node-right t))))]))
```

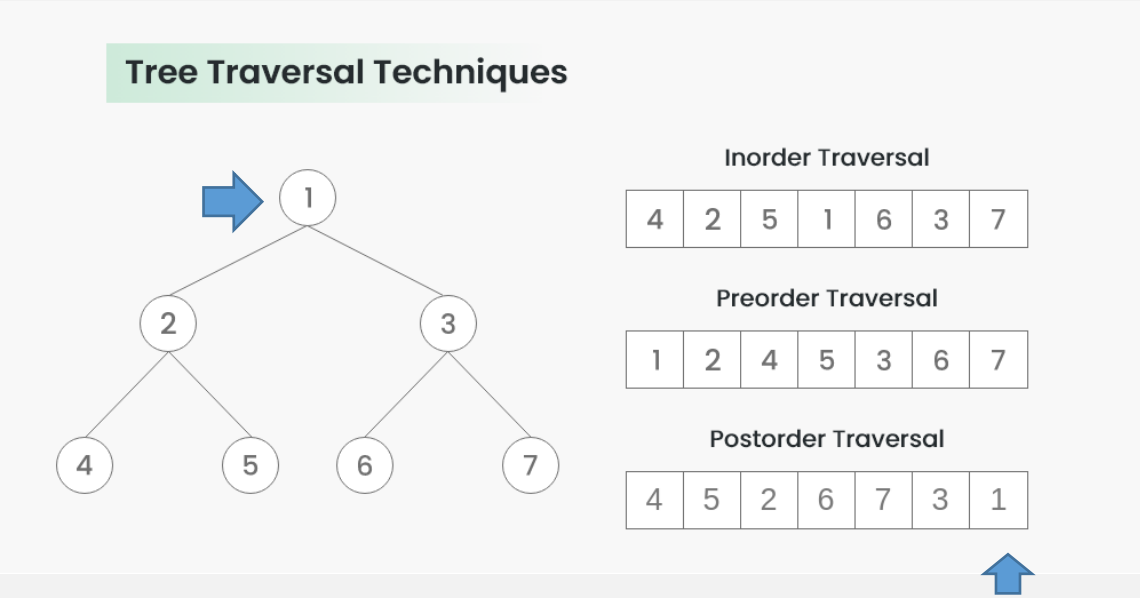
Pre-order Traversal




```
;; tree->lst/pre : Tree<X> -> List<X>
;; converts given tree to a list of values, by preorder
```

```
(define (tree->lst/pre t)
  (cond
    [(empty? t) empty]
    [(node? t) (cons (node-data t)
                     (append (tree->lst/pre (node-left t))
                             (tree->lst/pre (node-right t))))]))
```

Post-order Traversal




```
;; tree->lst/post : Tree<X> -> List<X>
;; converts given tree to a list of values, by postorder
```

```
(define (tree->lst/post t)
  (cond
    [(empty? t) empty]
    [(node? t) (append (tree->lst/post (node-left t))
                       (tree->lst/post (node-right t))
                       (list (node-data t)))]))
```

tree-all?

```
;; tree-all? : (X -> Boolean) Tree<X> -> Boolean
;; Returns true if given pred returns true
;; for all values in given tree
```

```
(define TREE1 (node empty 1 empty))
(define TREE3 (node empty 3 empty))
(define TREE123 (node TREE1 2 TREE3))
```

```
(check-true (tree-all? (curry < 4) TREE123))
```

Sometimes called andmap (for Racket lists) or every (for JS Arrays)

```
> (andmap positive? '(1 2 3))
#t
```

JavaScript Demo: Array.every()

```
1  const isBelowThreshold = (currentValue) => currentValue < 40;
2
3  const array1 = [1, 30, 39, 29, 10, 13];
4
5  console.log(array1.every(isBelowThreshold));
6  // Expected output: true
7
```

tree-all?

```
;; tree-all? : (X -> Boolean) Tree<X> -> Boolean
;; Returns true if given pred returns true
;; for all values in given tree
```

```
(define (tree-all? p? t)
  (cond
    [(empty? t) true]
    [(node? t)
     (and (p? (node-data t))
          (tree-all? p? (node-left t))
          (tree-all? p? (node-right t)))]))
```

Template:
cond clause for each itemization item

tree-all?

```
;; tree-all? : (X -> Boolean) Tree<X> -> Boolean
;; Returns true if given pred returns true
;; for all values in given tree
```

```
(define (tree-all? p? t)
  (cond
    [(empty? t) true]
    [(node? t)
     (and (p? (node-data t))
          (tree-all? p? (node-left t))
          (tree-all? p? (node-right t)))]))
```

tree-all?

```
;; tree-all? : (X -> Boolean) Tree<X> -> Boolean
;; Returns true if given pred returns true
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```

```
(define (tree-all? p? t)
  (cond
    [(empty? t) true]
    [(node? t)
     (and (p? (node-data t))
          (tree-all? p? (node-left t))
          (tree-all? p? (node-right t)))]))
```

Template:
Recursive call(s) match recursion in data definition

Template:
Extract pieces of compound data

tree-all?

```
;; tree-all? : (X -> Boolean) Tree<X> -> Boolean
;; Returns true if given pred returns true
;; for all values in given tree
```

```
(define (tree-all? p? t)
  (cond
    [(empty? t) true]
    [(node? t)
     (and (p? (node-data t))
          (tree-all? p? (node-left t))
          (tree-all? p? (node-right t)))]))
```

Combine the pieces with arithmetic to complete the function!

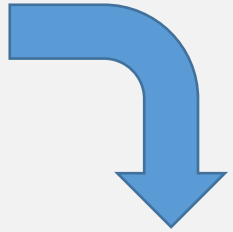
cond that evaluates to a boolean is just boolean arithmetic!

```
(define (tree-all? p? t)
  (or (empty? t)
      (and (p? (node-data t))
           (tree-all? p? (node-left t))
           (tree-all? p? (node-right t)))))
```

Tree Find?

Data Definitions With Invariants

```
;; A Tree<X> is one of:
;; - empty
;; - (node Tree<X> X Tree<X>)
(struct node [left data right])
;; a binary tree data structure
```



```
;; A BinarySearchTree<X> (BST) is a Tree<X>
;; where:
;; Invariant 1: for all values x in left tree, x < root val
;; Invariant 2: for all values y in right tree, y >= root val
```

Predicate?

Valid BSTs

```
;; valid-bst? : Tree<X> -> Bool
;; Returns true if the tree is a BST
```

```
(define TREE1 (node empty 1 empty))
(define TREE3 (node empty 3 empty))
(define TREE123 (node TREE1 2 TREE3))
```

```
(check-true (valid-bst? TREE123))
```

```
(check-false (valid-bst? (node TREE3 1 TREE2))
```

In-class Coding #3: Valid BST

Hint: use `tree-all?`

```
;; A BinarySearchTree<X> (BST) is a Tree<X>
;; where:
;; Invariant 1:
;; for all values x in left tree, x < root
;; Invariant 2:
;; for all values y in right tree, y >= root
```

```
;; valid-bst? : Tree<X> -> Bool
;; Returns true if the tree is a BST
```

```
(define TREE1 (node empty 1 empty))
(define TREE3 (node empty 3 empty))
(define TREE123 (node TREE1 2 TREE3))
```

```
(check-true (valid-bst? TREE123))
```

```
(check-false (valid-bst? (node TREE3 1 TREE2))
```

- `git add` **`bst-valid-`**<*your last name*>`.rkt`
  - E.g., `bst-valid-chang.rkt`
- `git commit bst-valid-chang.rkt -m 'add chang valid-bst?'`
- `git push origin main`
- Might need: `git pull --rebase`
  - If your local clone is not at `HEAD`

```
;; tree-fn : Tree<X> -> ???
(define (tree-fn t)
  (cond
    [(empty? t) ...]
    [(node? t) ... (tree-fn (node-left t)) ...
                   ... (node-data t) ...
                   ... (tree-fn (node-right t)) ...]))
```

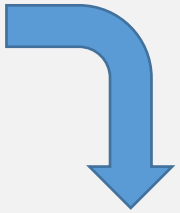
Valid BSTs

Hint: use `tree-all?`

```
;; valid-bst? : Tree<X> -> Bool
;; Returns true if the tree is a BST
```

cond that evaluates to a boolean is just boolean arithmetic!

```
(define (valid-bst? t)
  (cond
    [(empty? t) true]
    [(node? t)
     (and (tree-all? (curry > (node-data t)) (node-left t))
          (tree-all? (curry <= (node-data t)) (node-right t)))]))
```



```
(define (valid-bst? t)
  (or (empty? t)
      (and (tree-all? (curry > (node-data t)) (node-left t))
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```
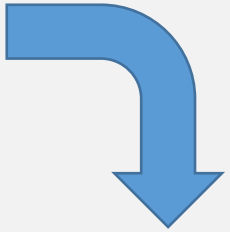

Data Definitions With Invariants

```
;; A Tree<X> is one of:
;; - empty
;; - (node Tree<X> X Tree<X>)
(struct node [left data right])
;; a binary tree data structure
```



"Deep" Invariants are enforced by individual functions

```
;; A BinarySearchTree<X> (BST) is a Tree<X>
;; where:
;; Invariant 1: for all values x in left tree, x < root val
;; Invariant 2: for all values y in right tree, y >= root val
```

```
(define (tree? x) (or (empty? x) (node? x)))
```

Predicate?

(For contracts, BST should use "shallow" `tree?` predicate, not "deep" `valid-bst?`)

BST Insert

Hint: use `valid-bst?` For tests

```
;; bst-insert : BST<X> X -> BST<X>
;; inserts given val into given bst, result is still a bst
```

```
(define TREE1 (node empty 1 empty))
(define TREE2 (node empty 2 empty))
(define TREE3 (node empty 3 empty))
(define TREE123 (node TREE1 2 TREE3))
```

```
(check-equal? (bst-insert (bst-insert TREE2 1) 3)
              TREE123))
```

```
(check-true (valid-bst? (bst-insert TREE123 4)))
```

In-class Coding #4: BST Insert

Hint: use `valid-bst?` For tests

```
;; A BinarySearchTree<X> (BST) is a Tree<X>
;; where:
;; Invariant 1:
;; for all values x in left tree, x < root
;; Invariant 2:
;; for all values y in right tree, y >= root
```

```
;; bst-insert : BST<X> X -> BST<X>
;; inserts given val into given bst,
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```

```
(define TREE1 (node empty 1 empty))
(define TREE2 (node empty 2 empty))
(define TREE3 (node empty 3 empty))
(define TREE123 (node TREE1 2 TREE3))
```

```
(check-equal? (bst-insert (bst-insert TREE2 1) 3) TREE123))
```

```
(check-true (valid-bst? (bst-insert TREE123 1)))
```

- `git add` **`bst-insert-`**<*your last name*>`.rkt`
  - E.g., `bst-insert-chang.rkt`
- `git commit bst-insert-chang.rkt -m 'add chang bst-insert'`
- `git push origin main`
- Might need: `git pull --rebase`
  - If your local clone is not at HEAD

```
;; tree-fn : Tree<X> -> ???
(define (tree-fn t)
  (cond
    [(empty? t) ...]
    [(node? t) ... (tree-fn (node-left t)) ...
                   ... (node-data t) ...
                   ... (tree-fn (node-right t)) ...]))
```

BST Insert

```
;; bst-insert : BST<X> X -> BST<X>
;; inserts given val into given bst, result is still a bst
```

```
(define (bst-insert bst x)
  (cond
    [(empty? bst) (node empty x empty)]
    [(node? bst)
     (if (< (node-data bst))
         (node (bst-insert (node-left t) x)
               (node-data t)
               (node-right t))
         (node (node-left t)
               (node-data t)
               (bst-insert (node-right t) x)))]))
```

Template:
cond clause for each itemization item

BST Insert

```
;; bst-insert : BST<X> X -> BST<X>
;; inserts given val into given bst, result is still a bst
```

```
(define (bst-insert bst x)
  (cond
    [(empty? bst) (node empty x empty)]
    [(node? bst)
     (if (< (node-data bst))
         (node (bst-insert (node-left t) x)
               (node-data t)
               (node-right t))
         (node (node-left t)
               (node-data t)
               (bst-insert (node-right t) x)))]))
```

BST Insert

```
;; bst-insert : BST<X> X -> BST<X>
;; inserts given val into given bst, result is still a bst
```

```
(define (bst-insert bst x)
  (cond
    [(empty? bst) (node empty x empty)]
    [(node? bst)
     (if (< (node-data bst))
         (node (bst-insert (node-left t) x)
               (node-data t)
               (node-right t))
         (node (node-left t)
               (node-data t)
               (bst-insert (node-right t) x)))]))
```

Template:
Recursive call matches recursion in data definition

Template:
Extract pieces of compound data

BST Insert

```
;; bst-insert : BST<X> X -> BST<X>
;; inserts given val into given bst, result is still a bst
```

```
(define (bst-insert bst x)
  (cond
    [(empty? bst) (node empty x empty)]
    [(node? bst)
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         (node (bst-insert (node-left t) x)
               (node-data t)
               (node-right t))
         (node (node-left t)
               (node-data t)
               (bst-insert (node-right t) x)))]))
```

Result must maintain **BST invariant!**

BST Insert

```
;; bst-insert : BST<X> X -> BST<X>
;; inserts given val into given bst, result is still a bst
```

```
(define (bst-insert bst x)
  (cond
    [(empty? bst) (node empty x empty)]
    [(node? bst)
     (if (< (node-data bst))
         (node (bst-insert (node-left t) x)
               (node-data t)
               (node-right t))
         (node (node-left t)
               (node-data t)
               (bst-insert (node-right t) x)))]))
```

Result must maintain **BST invariant!**

Smaller values on left

BST Insert

```
;; bst-insert : BST<X> X -> BST<X>
;; inserts given val into given bst, result is still a bst
```

```
(define (bst-insert bst x)
  (cond
    [(empty? bst) (node empty x empty)]
    [(node? bst)
     (if (< (node-data bst))
         (node (bst-insert (node-left t) x)
               (node-data t)
               (node-right t))
         (node (node-left t)
               (node-data t)
               (bst-insert (node-right t) x)))]))
```

Result must maintain **BST invariant!**

Larger values on right

Check-In Quiz 10/25

on gradescope

(due 1 minute before midnight)