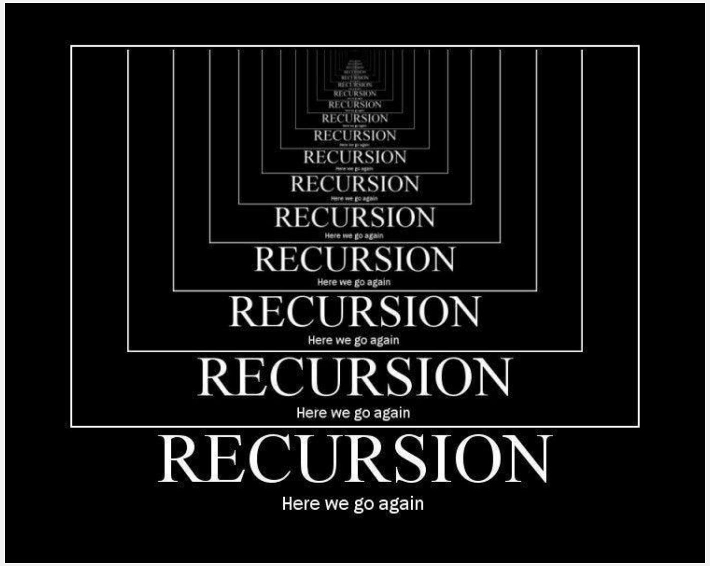
UMass Boston Computer Science

CS450 High Level Languages (section 2)

Generative Recursion

Wednesday, December 6, 2023

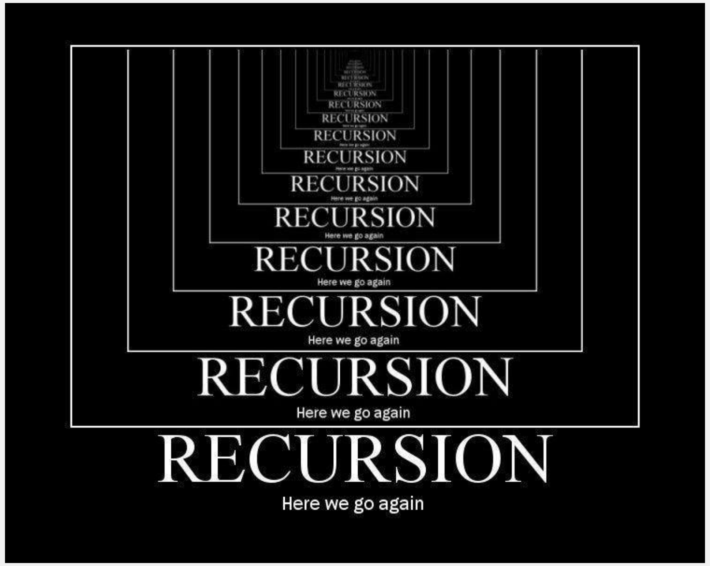


Logistics

- HW 9 out
  - <u>due</u>: Sun 12/10 11:59 pm EST

  - Starter code posted to `hw9-start` repo

(improper base case!)



Last Time

“bind/rec” in “CS450JS” Lang

```
;; A 450jsExpr (Expr) is one of:
;; …
;; - (list ‘bind/rec [Var Expr] Expr)
;; …
```

new binding <u>is in-scope</u> (can be referenced) **here**

Create new variable binding

new binding <u>is **also** in-scope</u> here!

Last Time

bind/rec examples

```
;; A 450jsExpr (Expr) is one of:
;; …
;; - (list 'bind/rec [Var Expr] Expr)
;; - (list 'iffy Expr Expr Expr)
;; …
```

JS “truthy” if (from hw7)

RACKET

```
(letrec
 ([fac
   (λ (n)
    (if (= n 0)
        1
        (* n (fac (- n 1)))))])
  (fac 5)) ; => 120
```

Equivalent to …

CS450JSLANG

```
(bind/rec
 [fac
  (fn (n)
   (iffy n
         (* n (fac (- n 1)))
         1))]
  (fac 5)) ; => 120
```

Zero is “truthy” false (hw7)

HW 9 Preview: Recursion!

Use "CS450JS LANG"! … to write recursive programs:

- `fac` (factorial)
- `filt` (list filter)
- `qsort` (functional quicksort)
- `gcd` (Euclid's algorithm)
- `sierpinski` (fractal triangle)

(Extra primitives will be added to `INIT-ENV`, ask if you need more)

- Look it up if you don't know any of these
  - Using any resources, e.g., ChatGPT, Co-pilot, is allowed
  - (still don't submit someone else's hw, obv)

Recursion review

- Most **recursion is structural** (comes from **data definitions**)!

```
(define (lst-fn lst)
  (cond
   [(empty? lst) …]
   [else … (first lst) … (lst-fn (rest lst)) …]))
```

TEMPLATE

```
;; A List<X> is
;; - empty
;; - (cons X List<X>)
```

A Different Kind of Recursion!

- Not all **recursion is structural** (comes from **data definitions**)!

A Different Kind of Recursion!

- <u>Not all </u>**recursion is structural** (comes from **data definitions**)!

```
;; gcd : Nat Nat -> Nat
;; computes greatest common divisor, using Euclid’s algorithm
;; termination argument:
;; m is halved (at least) eve                      ia modulo fn)
(define (gcd n m)
  (if (= m 0)
      n
      (gcd m (modulo n m))
```

What template is this following??

A Different Kind of Recursion!

- **Non-structural recursion** (doesn't come from **data definitions**) is called **generative recursion**
- no template? requires **Termination Argument**
  - Explains why the function terminates – bc recursive call is "smaller"!

```
;; gcd : Nat Nat -> Nat
;; computes greatest common divisor, using Euclid's algorithm
;; termination argument:
;; m is halved (at least) every iteration (via mo
(define (gcd n m)
  (if (= m 0)
      n
      (gcd m (modulo n m))))
```

But how to develop an algorithm like this??

Recursive call must be on "smaller" version of the problem

Generative (non-structural) Recursion Design Recipe

1. Name, Signature
2. Description
   - Must include **Termination Argument**
3. Examples
   - Even more important now!
4. Code (No structural template, but can use a “general” template)

5. Tests
6. Refactor

Generative (non-structural) Recursion Design Recipe

1. Name, Signature
2. Description
   - Must include **Termination Argument**
3. Examples
   - Even more important now!
4. Code (No structural template, but can use a "general" template)
   a) Break problems into <u>smaller</u> problems to (recursively) solve
   b) Determine how to combine smaller solutions
   c) "trivially solvable" problem is base case!
5. Tests
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Generative (non-structural) Recursion Design Recipe

4. Code (No structural template, but can use a “general” template)
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Generative (non-structural) Recursion Design Recipe

4. **Code** (No structural template, but can use a "general" template)
   a) **Break problems into smaller problems to (recursively) solve**
   b) Determine how to combine smaller solutions
   c) "trivially solvable" problem is base case!

```
;; genrec-algo: ??? -> ???
;; termination argument: recursive calls are "smaller" bc …
(define (genrec-algo problem)
  (cond
    [(trivial? problem) (solve-easy problem)] ;; base case
    [else (combine-solutions
           (genrec-algo (create-smaller-1 problem))
             …
           (genrec-algo (create-smaller-n problem)))]))
```

Generative (non-structural) Recursion Design Recipe

4. **Code** (No structural template, but can use a "general" template)
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```

GenRec Template Generalizes Structural!

- Trivial solution = data def base case
- Recursive smaller problem = data def smaller piece
- Left to figure out "Combining" pieces

```
(define (lst-fn lst)
  (cond
   [(empty? lst) …]
   [else … (first lst) … (lst-fn (rest lst)) …]))
```

```
;; genrec-algo: ??? -> ???

(define (genrec-algo problem)
  (cond
   [(trivial? problem) (solve-easy problem)] ;; base case
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               …
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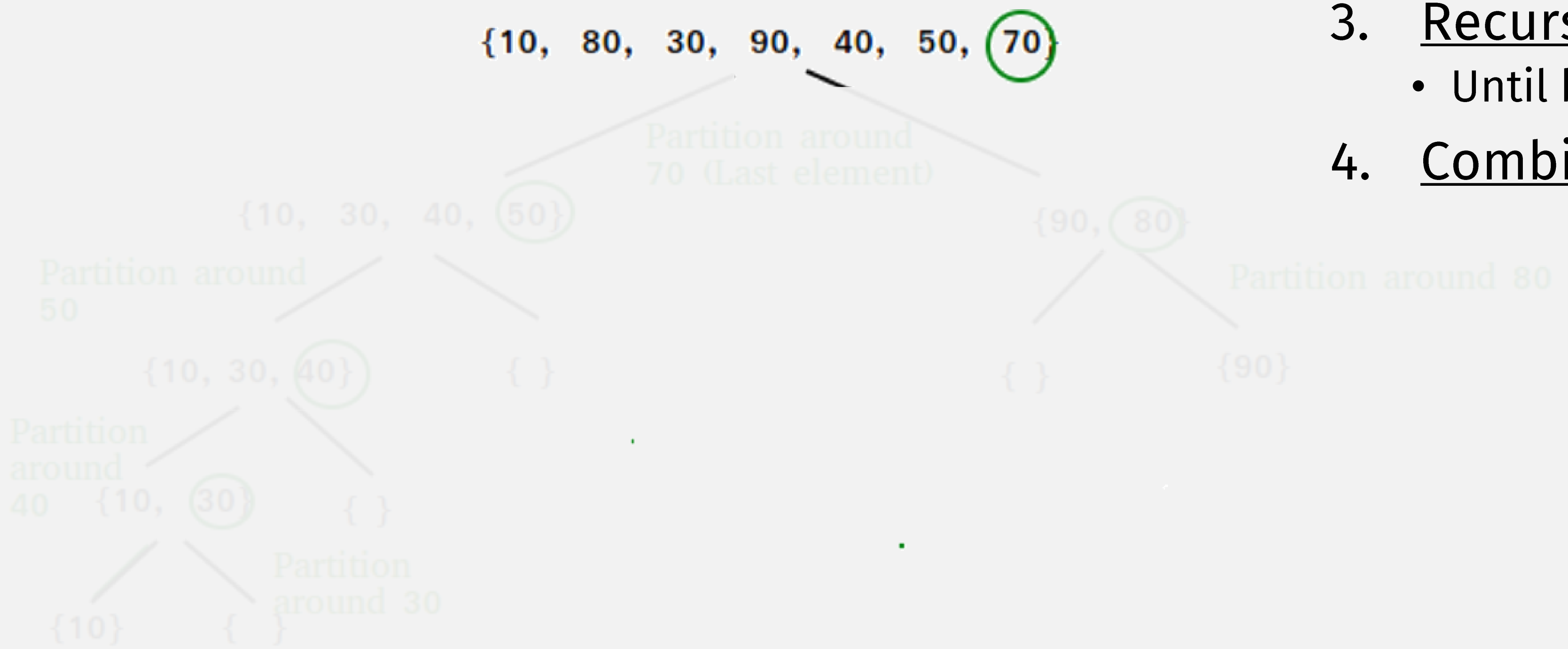
```

Gen Rec Example: (functional) quicksort

```
;; qsort: List<Int> -> List<Int>
;; termination argument: recursive calls are “smaller” bc …
(define (qsort lst)
  (cond
   [(trivial? problem) (solve-easy lst)] ;; base case
   [else (combine-solutions
           (qsort (create-smaller-1 lst))
               …
           (qsort (create-smaller-n lst)))]))
```

Quicksort overview ("divide and conquer")



1. Choose "pivot" element
2. Partition into smaller lsts:
   - < pivot
   - >= pivot
3. Recurse on smaller lists
   - Until base case
4. Combine small solutions

Gen Rec Example: (functional) quicksort

1. Choose "pivot" element
2. Partition into smaller lsts:
   - < pivot
   - >= pivot
3. Recurse until base case
4. Combine small solutions

```
;; qsort: List<Int> -> List<Int>
;; termination argument:
;; recursive calls drop at least pivot
(define (qsort lst)
  (cond
   [(trivial? problem) (solve-easy lst)] ;; base case
   [else
    (define pivot (first lst))
    (combine-solutions
      (qsort (smaller-problem-1 lst))
                …
            (qsort (smaller-problem-n lst)))]))
```

Gen Rec Example: (functional) quicksort

1. Choose “pivot” element
2. Partition into smaller lsts:
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;; qsort: List<Int> -> List<Int>
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(define (qsort lst)
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   [else
    (define pivot (first lst))
    (combine-solutions
      (qsort (filter (curry > pivot) lst))
                 …
            (qsort (filter (curry <= pivot) lst)))]))
```

Gen Rec Example: (functional) quicksort

1. Choose "pivot" element
2. Partition into smaller lsts:
   - < pivot
   - >= pivot
3. Recurse until base case
4. Combine small solutions

```
;; qsort: List<Int> -> List<Int>
;; termination argument:
;; recursive calls drop at least pivot
(define (qsort lst)
  (cond
   [(empty? lst) empty] ;; base case
   [else
    (define pivot (first lst))
    (combine-solutions
      (qsort (filter (curry > pivot) lst))
                   …
              (qsort (filter (curry <= pivot) lst)))]))
```

Gen Rec Example: (functional) quicksort

1. Choose "pivot" element
2. Partition into smaller lsts:
   - < pivot
   - >= pivot
3. Recurse until base case
4. Combine small solutions

```
;; qsort: List<Int> -> List<Int>
;; termination argument:
;; recursive calls drop at least pivot
(define (qsort lst)
  (cond
   [(empty? lst) empty] ;; base case
   [else
    (define pivot (first lst))
    (append
     (qsort (filter (curry > pivot) lst))
     (cons pivot
           (qsort (filter (curry <= pivot) lst))))]))
```

Gen Rec Example: (functional) quicksort

```
;; qsort: List<Int> -> List<Int>
;; termination argument:
;; recursive calls “smaller” bc at least one item dropped (pivot)
(define (qsort lst)
  (cond
   [(empty? lst) empty] ;; base case
   [else
    (define pivot (first lst))
    (append
      (qsort (filter (curry > pivot) lst))
      (cons pivot
            (qsort (filter (curry <= pivot) lst))))]))
```

Interlude: Recursion vs Iteration

- **Recursive** functions have a self-reference

```
def factorialUsingRecursion(n):
    if (n == 0):
        return 1;

    # recursion call
    return n * factorialUsingRecursion(n - 1);
```

- **Iterative** code typically use a loop

```
def factorialUsingIteration(n):
    res = 1;

    # using iteration
    for i in range(2, n + 1):
        res *= i;

    return res;
```

Recursion vs Iteration: Which is "Better"?

▸ **Recursive vs. Iterative Solutions**

Recursive algorithms can be very space inefficient. Each recursive call adds a new layer to the stack, which means that if your algorithm recurses to a depth of n, it uses at least O(n) memory.

For this reason, it's often better to implement a recursive algorithm iteratively. *All* recursive algorithms can be implemented iteratively, although sometimes the code to do so is much more complex. Before diving into recursive code, ask yourself how hard it would be to implement it iteratively, and discuss the tradeoffs with your interviewer.

Cracking the Coding Interview, Ch8

r/learnprogramming • 11 yr. ago

[Best Practices] Recursion. Why is it generally avoided and when is it acceptable?

stackoverflow

Are recursive methods always better than iterative methods in Java?

Recursion vs Iteration: Conventional Wisdom

Iteration

Strengths:

- Iteration can be used to repeatedly execute a set of statements without the overhead of function calls and without using stack memory.
- Iteration is faster and more efficient than recursion.
- It's easier to optimize iterative codes, and they generally have polynomial time complexity.
- They are used to iterate over the elements present in data structures like an array, set, map, etc.
- If the iteration count is known, we can use *for* loops; else, we can use *while* loops, which terminate when the controlling condition becomes false.

Weaknesses:

- In loops, we can go only in one direction, i.e., we can't go or transfer data from the current state to the previous state that has already been executed.
- It's difficult to traverse trees/graphs using loops.
- Only limited information can be passed from one iteration to another, while in recursion, we can pass as many parameters as we need.

Iteration is good with *non-recursive data*

Iteration is bad with *recursive data*!

Recursion better when **accumulators** are needed

https://www.interviewkickstart.com/learn/difference-between-recursion-and-iteration

Recursion vs Iteration: Conventional Wisdom

Recursion

Strengths:

- It's easier to code the solution using recursion when the solution of the current problem is dependent on the solution of smaller similar problems.
  - fibonacci(n) = fibonacci(n-1) + fibonacci(n-2)
  - factorial(n) = n * factorial(n-1)
- Recursive codes are smaller and easier to understand. ☑
- We can pass information to the next state in the form of parameters and return information to the previous state in the form of the return value.
- It's a lot easier to perform operations on trees and graphs using recursion.

Weaknesses:

- The simplicity of recursion comes at the cost of time and space efficiency.
- It is much slower than iteration due to the overhead of function calls and control shift from one function to another.
- It requires extra memory on the stack for each recursive call. This memory gets deallocated when function execution is over.
- is difficult to optimize a recursive code, and they generally have higher time complexity an iterative codes due to overlapping subproblems.

Recursion better when **accumulators** are needed

Use recursion with *recursive data*!

Recursion is slow

Recursion is slow

Recursion is slow

Recursion is slow

Investigate:

Is recursion is slower??

`https://www.interviewkickstart.com/learn/difference-between-recursion-and-iteration`

Recursion vs Iteration: In Racket

Racket Recursion

```
;; sum-to : Nat -> Nat
;; Sums the numbers in the interval [0, x]
(define (sum-to x)
  (if (zero? x)
      x
      (+ x (sum-to (sub1 x)))))
```

```
(define BIG-NUMBER 999999)

(time (sum-to BIG-NUMBER))
; cpu time: 202 real time: 201 gc time: 156
```

Conclusion?

Recursion is slower?

WAIT!

Racket does not have "for" loops

Racket "Iteration"

```
(time (for/sum ([x (add1 BIG-NUMBER)]) x))
; cpu time: 15 real time: 6 gc time: 0
```

Recursion vs Iteration: In Racket

Conclusion?
Recursion is <u>not</u> slower than iteration?

equivalent

Racket Recursion

```
;; iterative-sum-to : Nat -> Nat
;; Sums the numbers in the interval [0, x]
(define (iterative-sum-to x result)
  (if (zero? x)
      result
      (iterative-sum-to (sub1 x) (+ x result))))
```

accumulator

```
(time (iterative-sum-to BIG-NUMBER 0))
; cpu time: 15 real time: 13 gc time: 0
```

"for" in Racket is just a **macro** (i.e., "syntactic sugar") for a recursive function

Racket "Iteration"

```
(time (for/sum ([x (add1 BIG-NUMBER)]) x))
; cpu time: 15 real time: 6 gc time: 0
```

Tail Calls

From Wikipedia, the free encyclopedia

In computer science, a **tail call** is a subroutine call performed as the final action of a procedure. If the target of a tail is the same subroutine, the subroutine is said to be **tail recursive**, which is a special case of direct recursion. **Tail recursion** (or **tail-end recursion**) is particularly useful, and is often easy to optimize in implementations.

Tail calls can be implemented without adding a new stack frame to the call stack.

Recursion vs Iteration: In Racket

Racket Recursion

Conclusion?

Recursion is <u>not</u> slower than iteration?

```
;; iterative-sum-to : Nat -> Nat
;; Sums the numbers in the interval [0, x]
(define (iterative-sum-to x result)
  (if (zero? x)
      result
      (iterative-sum-to (sub1 x) (+ x result))))
```

Tail-recursive function

Tail-call (does not add to stack)

(Tail) recursion <u>is</u> iteration!

Recursion vs Iteration: Under the Hood

- **It makes sense that recursion** and **iteration** are **equivalent …**
  - Recursive call compiles to:
    - JUMP instruction
  - Loop compiles to:
    - JUMP instruction!

- **… except in languages that make them not equivalent!**
  - i.e., **languages that push extra stack frames that are not needed**

Tail-Calls in Other Languages

- **Most functional languages** (RACKET, HASKELL, ERLANG, F#) **implement proper tail calls** (no extra stack frame)

- Some languages require an explicit annotation
  - CLOJURE: `recur`
  - SCALA: `@tailrec`

- Most imperative languages don't properly implement tail calls (they add an unnecessary stack frame)
  - PYTHON, JAVA, C#, GO

Guido Got It Backwards

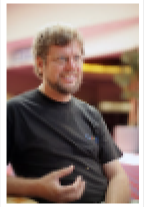
Wednesday, April 22, 2009

Tail Recursion Elimination

I recently posted an entry in my Python History blog on the origins of Python's functional features. A side remark about not supporting tail recursion elimination (TRE) immediately sparked several comments about what a pity it is that Python doesn't do this, including links to recent blog entries by others trying to "prove" that TRE can be added to Python easily. So let me defend my position (which is that I don't *want* TRE in the language). If you want a short answer, it's simply unpythonic. Here's the long answer:

First, as one commenter remarked, TRE is incompatible with nice stack traces: when a tail recursion is eliminated, there's no stack frame left to use to print a traceback when something goes wrong later. This will confuse users who inadvertently wrote something recursive (the recursion isn't obvious in the stack trace printed), and makes debugging hard. Providing an option to disable TRE seems wrong to me: Python's default is and should always be to be maximally helpful for debugging. This also brings me to the next issue:

About Me



Guido van Rossum

Python's BDFL

View my complete profile

Blog Archive

▶ 2022 (2)

▶ 2019 (1)

Wrong!

Equivalent to saying **every for loop iteration should push a stack frame!**

Proper tail calls is about **eliminating stack frames** that **shouldn't be there in the first place!** (because it's just iteration!)

Tail Calls as Loops

C

```c
int factorial(int n)
{
        int previous = 0xdeadbeef;

        if (n == 0 || n == 1) {
                return 1;
        }

        previous = factorial(n-1);
        return n * previous;
}

int main(int argc)
{
        int answer = factorial(5);
        printf("%d\n", answer);
}
```

Some languages directly compile recursion to a loop!
(with optimizations turned on)
(because they are equivalent!)

Proper Tail Calls in JavaScript

Proper Tail Calls (PTC) is a new feature in the ECMAScript 6 language. This feature was added to facilitate recursive programming patterns, both for direct and indirect recursion. Various other design patterns can benefit from PTC as well, such as code that wraps some functionality where the wrapping code directly returns the result of what it wraps. Through the use of PTC, the amount of memory needed to run code is reduced. In deeply recursive code, PTC enables code to run that would otherwise throw a stack overflow exception.

`https://webkit.org/blog/6240/ecmascript-6-proper-tail-calls-in-webkit/`

Not supported in V8 (Chrome) or SpiderMonkey (Firefox)!

Feature name	Compilers/polyfills				Desktop browsers								
	Babel 7 + core-js 3	Closure 2023.02	Type-Script + core-js 3	es6-shim	Konq 4.14[1]	IE 11	FF 115 ESR	FF 120	FF 121	FF 122	CH 116	CH 117	CH 118 Beta
	72%	55%	69%	17%	5%	11%	98%	98%	98%	98%	98%	98%	98%
proper tail calls (tail call optimisation)	0/2	0/2	0/2	0/2	0/2	0/2	0/2	0/2	0/2	0/2	0/2	0/2	0/2

Feature name	Desktop browsers										Servers/runtimes		
	CH 119 Dev	CH 120 Canary	Edge 113	Edge 114	SF 16.6	SF 17.0	SF TP	WK	OP 98	OP 99	Echo JS	XS6	JXA
	98%	98%	98%	98%	100%	100%	100%	99%	98%	98%	65%	94%	58%
proper tail calls (tail call optimisation)	0/2	0/2	0/2	0/2	2/2	2/2	2/2	2/2	0/2	0/2	0/2	2/2	0/2

Feature name	Servers/runtimes											Mobile		
	Node >=16.11 <17	Node >=18.3 <19	Node >=19.2 <20	Node >=20	DUK 2.7	JrS 2.4.0	JJS 1.8	GraalVM 21.3.3[5]	GraalVM 22.2.0[5]	Hermes 0.12.0	Deno 1.36	iOS 17.0	Samsung 22	Opera Mobile 77
	98%	98%	98%	98%	26%	96%	7%	98%	98%	74%	98%	100%	98%	98%
proper tail calls (tail call optimisation)	0/2	0/2	0/2	0/2	2/2	0/2	0/2	0/2	0/2	0/2	0/2	2/2	0/2	0/2

`https://compat-table.github.io/compat-table/es6/`

Recursion vs Iteration: Conclusion

Recursion

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Weaknesses:

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- It requires extra memory on the stack for each recursive call. This memory gets deallocated when function execution is over.
- It is difficult to optimize a recursive code, and they generally have higher time complexity than iterative codes due to overlapping subproblems.

Recursion is (usually) easier to read

Recursion better when **accumulators** are needed

Use recursion with *recursive data*!

Recursion is slower …

… in languages that choose to make it slower!

https://www.interviewkickstart.com/learn/difference-between-recursion-and-iteration

Fill Out Course Reviews!